SUBJECT NAME : Probability & Queueing Theory

SUBJECT CODE : MA 2262

MATERIAL NAME : University Questions

MATERIAL CODE : SKMA104

UPDATED ON : May – June 2013

Name of the Student: Branch:

Unit – I (Random Variables)

- ## Problems on Discrete & Continuous R.Vs

1. The distribution function of a random variable X is given by $F(X) = 1-(1+x)e^{-x};\ x \ge 0$. Find the density function, mean and variance of X. (N/D 2010)

2. A random variable X has the following probability function:

X	0	1	2	3	4	5	6	7
P(x)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(1) Find the value of k .

(2) Evaluate $p(X<6), p(X \ge 6)$

(3) If $p(X \le c) > \frac{1}{2}$ find the minimum value of c . (M/J 2012)

3. A random variable X has the following probability distribution

x	–2	–1	0	1	2	3
P(x)	0.1	K	0.2	2k	0.3	3k

(1) Find k , (2) Evaluate $p(X<2)$ and $p(-2<X<2)$, (3) Find the PDF of X and (4) Evaluate the mean of X . (N/D 2011)

4. If the random variable X takes the values 1,2,3 and 4 such that $2P(X=1)=3P(X=2)=P(X=3)=5P(X=4)$, then find the probability distribution and cumulative distribution function of X . (N/D 2012)

5. If X is a random variable with a continuous distribution function $F(X)$, prove that $Y = F(X)$ has a uniform distribution in $(0,1)$. Further if

$$f(X)=\begin{cases}\frac{1}{2}(x-1); & 1\le x\le 3\\ 0; & \text{otherwise}\end{cases},$$

find the range of Y corresponding to the range $1.1 \le x \le 2.9$. (N/D 2010)

6. If the density function of X equals $f(x)=\begin{cases}Ce^{-2x}, & 0<x<\infty\\ 0, & x<0\end{cases}$, find C. What is $P[X>2]$? (A/M 2010)

7. A continuous random ariable has the pdf $f(x)=kx^4,\quad -1<x<0$. Find the value of k and also $p\left\{X>\left(\frac{-1}{2}\right)/X<\left(\frac{-1}{4}\right)\right\}$. (M/J 2013)

8. The DF of a continuous random variable X is given by

$$F(x)=\begin{cases}0, & x<0\\ x^2, & 0\le x\le 1\\ 1-\frac{3}{25}(3-x^2), & \frac{1}{2}\le x\le 3\\ 1, & x\ge 3\end{cases}$$

. Find the pdf of X and evaluate $p(X\le 1)$ and $p\left(\frac{1}{3}<X<4\right)$ using both the CDF and PDF. (N/D 2011)

9. The probability function of an infinite discrete distribution is given by $p(X=j)=\frac{1}{2^j};\ j=1,2\ldots,\infty$. Verify that the total probability is 1 and find the mean and variance of the distribution. Find also $p(X \text{ is even}),\ p(X\ge 5)$ and $p(X \text{ is divisible by } 3)$. (N/D 2011)

- **Moments and Moment Generating Function**

1. Find the MGF of the binomial distribution and hence find its mean. (N/D 2012)

2. Find the moment-generating function of the binomial random variable with parameters m and p and hence find its means and variance. (A/M 2011)

3. Find the moment generating function of Uniform distribution Hence find its mean and variance. (M/J 2013)

4. Define Gamma distribution and find its mean and variance. (N/D 2011)

5. Find the moment generating function of an exponential random variable and hence find its mean and variance. (M/J 2012)

6. Find the moment-generating function of the binomial random variable with parameters m and p . Hence find its means and variance. (A/M 2011)

7. Define Weibull distribution and write its mean and variance. (A/M 2011)

8. Describe the situations in which geometric distributions could be used. Obtain its moment generating function. (A/M 2010)

9. By calculating the moment generation function of Poisson distribution with parameter λ , prove that the mean and variance of the Poisson distribution are equal.(A/M 2010)

10. Derive mean and variance of a Geometric distribution. Also establish the forgetfulness property of the Geometric distribution. (A/M 2011)

11. Find the moment generating function and rth moment for the distribution whose pdf is $f(x) = ke^{-x}, \quad 0 \le x \le \infty$. Hence find the mean and variance. (M/J 2013)

- **Problems on distributions**

1. If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test (1) on the 4th trial (2) in fewer than 4 trials? (N/D 2012)

2. A coin is tossed until the first head occurs. Assuming that the tosses are independent and the probability of a head occurring is 'p'. Find the value of 'p' so that the probability that an odd number of tosses required is equal to 0.6. Can you find a value of 'p' so that the probability is 0.5 that an odd number of tosses are required? (N/D 2010)

3. The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$ .What is the probability that the repair time exceeds 2h? What is the conditional probability that a repair takes at least 10h given that its duration exceeds 9h? (N/D 2010)

4. A coin having probability p of coming up heads is successively flipped until the r[th] head appears. Argue that X, the number of flips required will be n, $n \geq r$ with probability

$P[X = n] = \binom{n-1}{r-1} p^r q^{n-r}$, $n \geq r$. (A/M 2010)

5. Suppose that telephone calls arriving at a particular switchboard follow a Poisson process with a an average of 5 calls coming per minute. What is the probability that up to a minute will elapse unit 2 calls have come in to the switch board? (A/M 2011)

6. The number of monthly breakdowns of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month (1) without a breakdown (2) with only one breakdown. (N/D 2012)

7. In a large consignment of electric bulbs, 10 percent are defective. A random sample of 20 is taken for inspection. Find the probability that (1) all are good bulbs (2) at most there are 3 defective bulbs (3) exactly there are 3 defective bulbs. (M/J 2013)

8. If X is a Poisson variate such that $p(X = 2) = 9p(X = 4) + 90p(X = 6)$. Find

   (1) Mean and $E(X^2)$ (2) $p(X \geq 2)$ (M/J 2012)

9. In a certain city, the daily consumption of electric power in millions of kilowatt hours can be treated as a random variable having Gamma distribution with parameters $\lambda = \frac{1}{2}$ and $v = 3$. If the power plant of this city has a daily capacity of 12 millions kilowatt-hours, what is the probability that this power supply will be inadequate on any given day? (M/J 2012)

Unit – II (Two Dimensional Random Variables)

• Joint distributions – Marginal & Conditional

1. Given $f(x, y) = cx(x - y),\ 0 < x < 2,\ -x < y < x$ and '0' elsewhere. Evaluate 'c' and find $f_X(x)$ and $f_Y(y)$ respectively. Compute $P[X < Y]$. (N/D 2010)

2. The joint probability mass function of (X, Y) is given by $p(x, y) = K(2x + 3y)$, $x = 0,1,2;\ y = 1,2,3$. Find all the marginal and conditional probability distributions. (N/D 2011)

3. Let X and Y be two random variables having the joint probability function $f(x, y) = k(x + 2y)$ where x and y can assume only the integer values 0, 1 and 2. Find the marginal and conditional distributions. (M/J 2012)

4. Suppose that X and Y are independent non negative continuous random variables having densities $f_X(x)$ and $f_Y(y)$ respectively. Compute $P[X < Y]$. (A/M 2010)

5. The joint density of X and Y is given by $f(x,y) = \begin{cases} \frac{1}{2} y e^{-xy}, & x > 0, 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$. Calculate the conditional density of X given Y = 1. (A/M 2010)

6. The joint pdf of random variable X and Y is given by $f(x,y) = \begin{cases} \lambda x y^2, & 0 \le x \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$.
(1) Determine the value of λ. (2) Find the marginal probability density function of X. (N/D 2012)

7. Given the joint density function $f(x,y) = \begin{cases} x \frac{(1+3y^2)}{4}, & 0 < x < 2, 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$. Find the marginal densities $g(x), h(y)$ and the conditional density $f(x/y)$ and evaluate $P\left[\frac{1}{4} < x < \frac{1}{2} / Y = \frac{1}{3}\right]$. (A/M 2011)

8. Determine whether the random variables X and Y are independent, given their joint probability density function as $f(x,y) = \begin{cases} x^2 + \frac{xy}{3}, & 0 \le x \le 1,\ 0 \le y \le 2 \\ 0, & \text{otherwise} \end{cases}$.
(A/M 2011)

9. The joint probability density function of a two-dimensional random variable (X,Y) is $f(x,y) = \frac{1}{8}(6 - x - y),\ 0 < x < 2,\ 2 < y < 4$. Find (1) $P(X < 1 \cap Y 3)$
(2) $P(X + Y < 3)$ (3) $P(X < 1 / Y < 3)$ (M/J 2013)

10. Two dimensional random variables (X,Y) have the joint probability density function
$$f(x,y) = 8xy,\quad 0 < x < y < 1$$
$$= 0, \text{elsewhere}$$.
(1) Find $P\left(X < \frac{1}{2} \cap Y < \frac{1}{4}\right)$.
(2) Find the marginal and conditional distributions.

(3) Are X and Y independent? (M/J 2012)

- **Covariance, Correlation and Regression**

1. For two random variable X and Y with the same mean, the two regression equations are $y = ax + b$ and $x = cy + d$. Find the common mean, ratio of the standard deviations and also show that $\frac{b}{d} = \frac{1-a}{1-c}$. (N/D 2010)

2. The regression equation of X and Y is $3y - 5x + 108 = 0$. If the mean value of Y is 44 and the variance of X were 9/16 th of the variance of Y. Find the mean value of X and the correlation coefficient. (N/D 2012)

3. If the correlation coefficient is 0, then can we conclude that they are independent? Justify your answer, through an example. What about the converse? (A/M 2010)

4. Two random variables X and Y have the joint probability density function $f(x, y) = \begin{cases} c(4 - x - y), & 0 \le x \le 2,\ 0 \le y \le 2 \\ 0, & \text{elsewhere} \end{cases}$. Find $\text{cov}(X, Y)$. (M/J 2012)

5. Obtain the equations of the lines of regression from the following data: (N/D 2012)

X:	1	2	3	4	5	6	7
Y:	9	8	10	12	11	13	14

6. Find the correlation coefficient for the following data: (N/D 2011)

X	10	14	18	22	26	30
Y	18	12	24	6	30	36

7. The marks obtained by 10 students in Mathematics and Statistics are given below. Find the correlation coefficient between the two subjects. (M/J 2013)

Marks in maths	75	30	60	80	53	35	15	40	38	48
Marks in Stats.	85	45	54	91	58	63	35	43	45	44

8. Compute the coefficient of correlation between X and Y using the following data: (N/D 2010)

X:	1	3	5	7	8	10
Y:	8	12	15	17	18	20

- **Transformation of the random variables**

1. If X and Y are independent RVs with pdf's $e^{-x}, x \ge 0,$ and $e^{-y}, y \ge 0,$ respectively, find the density functions of $U = \frac{X}{X+Y}$ and $V = X + Y$. Are U and V independent?

(N/D 2011)

2. If X and Y each follow an exponential distribution with parameter 1 and are independent, find the pdf of $U = X - Y$. (M/J 2013)

3. Let X and Y be independent random variables both uniformly distributed on $(0,1)$. Calculate the probability density of $X + Y$. (A/M 2010)

4. If X and Y are independent random variables having density functions $f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ and $f(y) = \begin{cases} 3e^{-3y}, & y \geq 0 \\ 0, & y < 0 \end{cases}$, respectively, find the density functions of $z = X - Y$. (A/M 2011)

- ## Central Limit Theorem

1. State and prove central limit theorem. (N/D 2011)

2. If $X_1, X_2, \ldots X_n$ are Poisson variates with parameter $\lambda = 2$, use the central limit theorem to estimate $P(120 \leq S_n \leq 160)$, where $S_n = X_1 + X_2 + \ldots + X_n$ and $n = 75$. (N/D 2010)

3. Let $X_1, X_2, \ldots X_{100}$ be independent identically distributed random variables with $\mu = 2$ and $\sigma^2 = \frac{1}{4}$. Find $P(192 < X_1 + X_2 + \ldots + X_{100} < 210)$. (N/D 2012)

4. Suppose that in a certain circuit, 20 resistors are connected in series. The mean and variance of each resistor are 5 and 0.20 respectively. Using Central limit theorem, find the probability that the total resistance of the circuit will exceed 98 ohms assuming independence. (M/J 2012)

5. A distribution with unknown mean μ has variance equal to 1.5. Use central limit theorem to find how large a sample should be taken from the distribution in order that the probability will be at least 0.95 that the sample mean will be within 0.5 of the population mean. (M/J 2013)

Unit – III (Markov Processes and Markov Chains)

- ## Verification of SSS and WSS process

1. Show that the random process $X(t) = A\cos(\omega_0 t + \theta)$ is wide – sense stationary, if A and ω_0 are constants and θ is uniformly distributed RV in $(0, 2\pi)$. (N/D 2011)

2. Show that the process $X(t) = A\cos\lambda t + B\sin\lambda t$ is wide sense stationary, if $E(\mathrm{A}) = \mathrm{E(B)} = 0,\ \mathrm{E(A^2)} = \mathrm{E(B^2)}$ and $\mathrm{E(AB)} = 0$, where A and B are random variables. (M/J 2013)

3. Show that random process $\{X(t)\} = A\cos t + B\sin t,\ -\infty < t < \infty$ is a wide sense stationary process where A and B are independent random variables each of which has a value -2 with probability $\frac{1}{3}$ and a value 1 with probability $\frac{2}{3}$. (A/M 2011)

4. The process $\{X(t)\}$ whose probability distribution under certain condition is given by

$$P\left[X(t)=n\right] = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1,2,3,\ldots \\ \dfrac{at}{1+at}, & n = 0 \end{cases}$$

. Show that $\{X(t)\}$ is not stationary.

(M/J 2012)

• Problems on Markov Chain

1. An engineer analyzing a series of digital signals generated by a testing system observes that only 1 out of 15 highly distorted signals follow a highly distorted signal, with no recognizable signal between, whereas 20 out of 23 recognizable signals follow recognizable signals, with no highly distorted signal between. Given that only highly signals are not recognizable, find the fraction of signals that are highly distorted. (N/D 2010)

2. An observer at a lake notices that when fish are caught, only 1 out of 9 trout is caught after another trout, with no other fish between, whereas 10 out of 11 non-trout are caught following non-trout, with no trout between. Assuming that all fish are equally likely to be caught, what fraction of fish in the lake is trout? (N/D 2012)

3. A man either drives a car (or) catches a train to go to office each day. He never goes two days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared. Find (1) the probability that he takes a train on the third day and (2) the probability that he drives to work in the long run. (N/D 2011)

4. A salesman territory consists of three cities A, B and C. He never sells in the same city on successive days. If he sells in city-A, then the next day he sells in city-B. However if he sells in either city-B or city-C, the next day he is twice as likely to sell in city-A as in the other city. In the long run how often does he sell in each of the cities? (M/J 2012)

5. A fair die is tossed repeatedly. The maximum of the first 'n' outcomes is denoted by X_n. Is $\{X_n : n = 1, 2, \ldots\}$ a Markov chain? Why or why not? If it is a Markov chain, calculate its transition probability matrix. Specify the classes. (N/D 2012)

6. Suppose that a mouse is moving inside the maze shown in the adjacent figure from one cell to another, in search of food. When at a cell, the mouse will move to one of the adjoining cells randomly. For $n \geq 0,\ X_n$ be the cell number the mouse will visit after having changed cells 'n' times. Is $\{X_n ; n = 0, 1, \ldots\}$ a Markov chain? If so, write its state space and transition probability matrix. (N/D 2010)

1	4	7
2	5	8
3	6	9

7. Suppose that whether or not it rains today depends on previous weather conditions through the last two days. Show how this system may be analyzed by using a Markov chain. How many states are needed? (A/M 2010)

8. Three out of every four trucks on the road are followed by a car, while only one out of every five cars is followed by a truck. What fraction of vehicles on the road are trucks? (A/M 2010)

9. Let the Markov Chain consisting of the states 0, 1, 2, 3 have the transition probability matrix. $P = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$ Determine which states are transient and which are recurrent by defining transient and recurrent states. (A/M 2010)

10. Find the limiting-state probabilities associated with the following transition probability matrix. $\begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}$. (A/M 2011)

11. The transition probability matrix of a Markov chain $\{X(t)\},\ n = 1, 2, 3, \ldots$, having three states 1, 2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $p^{(0)} = (0.7 \quad 0.2 \quad 0.1)$. Find (1) $p[X_2 = 3]$ (2) $p\ X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2]$. (M/J 2012)

12. The following is the transition probability matrix of a Markov chain with state space $\{1, 2, 3, 4, 5\}$. Specify the classes, and determine which classes are transient and which are recurrent. $P = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1/3 & 0 & 2/3 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2/5 & 0 & 3/5 \end{pmatrix}$ (N/D 2012)

13. Find the nature of the states of the Markov chain with the tpm $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$. (M/J 2013)

14. The following is the transition probability matrix of a Markov chain with state space $\{0, 1, 2, 3, 4\}$. Specify the classes, and determine which classes are transient and which are recurrent. Give reasons. $P = \begin{pmatrix} 2/5 & 0 & 0 & 3/5 & 0 \\ 1/3 & 1/3 & 0 & 1/3 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 1/4 & 0 & 0 & 3/4 & 0 \\ 0 & 0 & 1/3 & 0 & 2/3 \end{pmatrix}$ (N/D 2010)

15. A gambler has Rs.2. He bets Rs.1 at a time and wins Rs.1 with probability $1/2$. He stops playing if he loses Rs.2 or wins Rs.4. (1) What is the tpm of the related Markov chain? (2) What is the probability that he has lost his money at the end of 5 plays? (M/J 2013)

• Poisson process

1. Define Poisson process and derive the Poisson probability law. (N/D 2011)

2. Derive probability distribution of Poisson process and hence find its auto correlation function. (A/M 2011)

3. Show that the difference of two independent Poisson processes is not a Poisson process. (A/M 2011),(M/J 2013)

4. Prove that the Poisson process is a Markov process. (M/J 2013)

5. If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2/min, find the probability that the interval between 2 consecutive arrivals is more than 1 min, between 1 and 2 mins, and 4 mins or less. (N/D 2010)

6. If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is (1) more than 1 min. (2) between 1 min and 2 min and (3) 4 min (or) less. (N/D 2011)

7. Customers arrive at a one window drive-in bank according to Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The space is front of window, including that for the serviced car can accommodate a maximum of three cars. Others cars can wait outside this space. (A/M 2011)

   (1) What is the probability that an arriving customer can drive directly to the space in front of the window?

   (2) What is the probability that an arriving customer will have to wait outside the indicated space?

   (3) How long is an arriving customer expected to wait before being served?

8. Suppose that customers arrive at a bank according to Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of two minutes

   (1) exactly 4 customers arrive

   (2) greater than 4 customers arrive

   (3) fewer than 4 customers arrive. (M/J 2012)

Unit – IV (Queueing Theory)

• Model – I (M/M/1) : (∞/FIFO)

1. Customer arrive at a one man barber shop according to a Poisson process with a mean inter arrival time of 20 minutes. Customers spend an average of 15 minutes in the barber chair. The service time is exponentially distributed. If an hour is used as a unit of time, then (M/J 2013)

    (i) What is the probability that a customer need not wait for a haircut?
    (ii) What is the expected number of customer in the barber shop and in the queue?
    (iii) How much time can a customer expect to spend in the barber shop?
    (iv) Find the average time that a customer spend in the queue.
    (v) Estimate the fraction of the day that the customer will be idle?
    (vi) What is the probability that there will be 6 or more customers?
    (vii) Estimate the percentage of customers who have to wait prior to getting into the barber's chair.

2. If people arrive to purchase cinema tickets at the average rate of 6 per minute, it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 min before the picture starts and it takes exactly 1.5 min to reach the correct seat after purchasing the ticket, (N/D 2010)

    (i) Can he expect to be seated for the start of the picture?
    (ii) What is the probability that he will be seated for the start of the picture?
    (iii) How early must he arrive in order to be 99% sure of being seated for the start of the picture?

3. A T.V. repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If he repair sets in the order in which they came in and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 hour day. Find (M/J 2012)

    (1) the repairman's expected idle time each day
    (2) how many jobs are ahead of average set just brought?

• Model – II (M/M/C) : (∞/FIFO)

1. There are 3 typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour,
   - (i) What fraction of the time all the typists will be busy?
   - (ii) What is the average number of letters waiting to be typed?
   - (iii) What is the average time a letter has to spend for waiting and for being typed? (N/D 2010),(N/D 2011)

2. A supermarket has 2 girls running up sales at the counters. If the service time for each customer is exponential with mean 4 minutes and if people arrive in Poisson fashion at the rate of 10 per hour, find the following: (M/J 2012)
   - (1) What is the probability of having to wait for service?
   - (2) What is the expected percentage of idle time for each girl?
   - (3) What is the expected length of customer's waiting time?

3. Trains arrive at the yard every 15 minutes and the service time is 33 minutes. If the line capacity of the yard is limited to 5 trains, find the probability that the yard is empty and the average number of trains in the system, given that the inter arrival time and service time are following exponential distribution. (M/J 2012)

4. There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour, what fraction of times all the typists will be busy? What is the average number of letters waiting to be typed? (M/J 2012),(M/J 2013)

• Model – III (M/M/1) : (K/FIFO)

1. At a port there are 6 unloading berths and 4 unloading crews. When all the berths are full, arriving ships are diverted to an overflow facility 20 kms down the river. Tankers arrive according to Poisson process with a mean of 1 every 2 hrs. It takes for an unloading crew, on the average, 10 hrs to unload a tanker, the unloading time following an exponential distribution. Find
   - (i) how many tankers are at the port on the average?
   - (ii) how long does a tanker spend at the port on the average?
   - (iii) what is the average arrival rate at the overflow facility? (N/D 2012)

• Derivations of Queueing Models

1. Define birth and death process. Obtain its steady state probabilities. How it could be used to find the steady state solution for the M/M/1 model? Why is it called geometric? (A/M 2010)

2. Obtain the steady state probabilities of birth-death process. Also draw the transition graph. (N/D 2012)

3. Derive (1) L_s , average number of customers in the system (2) L_q , average number of customers in the queue for the queuing model (M/M/1):(N/FIFO). (M/J 2013)

4. Calculate any four measures of effectiveness of M/M/1 queueing model. (A/M 2010)

5. Show that for the $\left(M/M/1\right):\left(FCFS/\infty/\infty\right)$, the distribution of waiting time in the system is $w(t)=\left(\mu-\lambda\right)e^{-(\mu-\lambda)t}, \ \ t>0$. (A/M 2011)

6. Find the steady state solution for the multiserver M/M/C model and hence find L_q, W_q, W_s and L_s by using Little formula. (A/M 2011)

7. Find the mean number of customers in the queue, system, average waiting time in the queue and system of @ queueing model. (N/D 2011)

Unit – V (Non – Markovian Queues and Queue Networks)

• Pollaczek – Khinchine formula

1. Derive Pollaczek – Khintchine formula of M/G/1 queue.
(N/D 2010),(N/D 2011), (A/M 2010),(M/J 2012),(N/D 2012)

2. Derive the expected steady state system size for the single server queues with Poisson input and General service. (A/M 2011)

3. Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. The parking lot is large enough to accommodate any number of cars. If the service time for all cars is constant and equal to 10 minutes, determine (M/J 2012),(M/J 2013)

   (1) mean number of customers in the system L_s

(2) mean number of customers in the queue L_q

(3) mean waiting time of a customer in the system W_s

(4) mean waiting time of a customer in the queue W_q

- ## Queueing networks

1. Explain how queueing theory could be used to study computer networks. (A/M 2010)
2. Consider a two stage tandem queue with external arrival rate λ to node '0'. Let μ_0 and μ_1 be the service rates of the exponential servers at node '0' and '1' respectively. Arrival process is Poisson. Model this system using a Markov chain and obtain the balance equations. (N/D 2012)
3. Write short notes on the following :

   (i) Queue networks

   (ii) Series queues

   (iii) Open networks

   (iv) Closed networks (N/D 2010),(A/M 2011)

4. Discuss open and closed networks. (N/D 2011)

5. Consider two servers. An average of 8 customers per hour arrive from outside at server 1 and an average of 17 customers per hour arrive from outside at server 2. Inter arrival times are exponential. Server 1 can serve at an exponential rate of 20 customers per hour and server 2 can serve at an exponential rate of 30 customers per hour. After completing service at station 1, half the customers leave the system and half go to server 2. After completing service at station 2, 3/4 of the customer complete service and 1/4 return to server 1. Find the expected no. of customers at each server. Find the average time a customer spends in the system. (N/D 2012)

6. Consider a system of two servers where customers from outside the system arrive at server 1 at a Poisson rate 4 and at server 2 at a Poisson rate 5. The service rates for server 1 and 2 are 8 and 10 respectively. A customer upon completion of service at server 1 is likely to go to server 2 or leave the system; whereas a departure from server 2 will go 25 percent of the time to server 1 and will depart the system otherwise. Determine the limiting probabilities, L_s and W_s . (M/J 2013)

7. An average of 120 students arrive each hour (inter-arrival times are exponential) at the controller office to get their hall tickets. To complete the process, a candidate must pass through three counters. Each counter consists of a single server, service times at each counter are exponential with the following mean times: counter 1, 20 seconds; counter 2, 15 seconds and counter 3, 12 seconds. On the average how many students will be present in the controller's office. (M/J 2012)

8. For a open queueing network with three nodes 1, 2 and 3, let customers arrive from outside the system to node j according to a Poisson input process with parameters r_j and let P_{ij} denote the proportion of customers departing from facility i to facility j.
Given $(r_1, \; r_2, \; r_3) = (1, \; 4, \; 3)$ and $P_{ij} = \begin{bmatrix} 0 & 0.6 & 0.3 \\ 0.1 & 0 & 0.3 \\ 0.4 & 0.4 & 0 \end{bmatrix}$ determine the average arrival rate λ_j to the node j for $j = 1, 2, 3$. (M/J 2012)

----All the Best----