SUBJECT NAME : Probability & Random Process

SUBJECT CODE : MA 2261

MATERIAL NAME : University Questions

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Name of the Student: Branch:

Unit – I (Random Variables)

• Problems on Discrete & Continuous R.Vs

1. A random variable X has the following probability distribution.

X	0	1	2	3	4	5	6	7
P(x)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find:

(1) The value of k

(2) $P(1.5 < X < 4.5 / X > 2)$ and

(3) The smallest value of n for which $P(X \le n) > \frac{1}{2}$.

(N/D 2010),(M/J 2012)

2. The probability mass function of random variable X is defined as $P(X = 0) = 3C^2$, $P(X = 1) = 4C - 10C^2$, $P(X = 2) = 5C - 1$, where $C > 0$ and $P(X = r) = 0$ if $r \ne 0,1,2$. Find

(1) The value of C

(2) $P(0 < X < 2 / x > 0)$

(3) The distribution function of X

(4) The largest value of X for which $F(x) < \frac{1}{2}$. (A/M 2010)

3. The probability density function of a random variable X is given by

$$f_X(x) = \begin{cases} x, & 0 < x < 1 \\ k(2 - x), & 1 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}.$$

(1) Find the value of 'k'.

(2) Find $P(0.2 < x < 1.2)$

(3) What is $P[0.5 < x < 1.5 / x \ge 1]$

(4) Find the distribution function of $f(x)$. (A/M 2011)

4. A continuous R.V. X has the p.d.f. $f(x) = \begin{cases} \frac{k}{1 + x^2}, & -\infty < x < \infty \\ 0, & \text{elsewhere} \end{cases}$. Find

(1) the value of k

(2) Distribution function of X

(3) $P(X \ge 0)$ (N/D 2011)

5. Show that for the probability function $p(x) = P(X = x) = \begin{cases} \frac{1}{x(x+1)}, & x = 1, 2, 3... \\ 0, & \text{otherwise} \end{cases}$, $E(X)$ does not exist. (N/D 2012)

6. The probability function of an infinite discrete distribution is given by

$P(X = j) = \frac{1}{2^j}$ $(j = 1, 2, 3, ...)$ Find

(1) Mean of X

(2) $P(X \text{ is even})$ and

(3) $P(X \text{ is divisible by } 3)$ (N/D 2011)

• Moments and Moment Generating Function

1. Find the MGF of the two parameter exponential distribution whose density function is given by $f(x) = \lambda e^{-\lambda(x-a)}$, $x \ge a$ and hence find the mean and variance.

(A/M 2010)

2. Derive the m.g.f of Poisson distribution and hence or otherwise deduce its mean and variance.(A/M 2011)

3. If the probability density of X is given by $f(x) = \begin{cases} 2(1-x) & \text{for } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$, find its rth moment. Hence evaluate $E\left[\left(2X+1\right)^2\right]$. (N/D 2012)

4. Find the M.G.F. of the random variable X having the probability density function $f(x) = \begin{cases} \frac{x}{4} e^{-2x}, & x>0 \\ 0, & \text{elsewhere} \end{cases}$. Also deduce the first four moments about the origin.

(N/D 2010),(M/J 2012)

5. Find MGF corresponding to the distribution $f(\theta) = \begin{cases} \frac{1}{2} e^{-2\theta}, & \theta > 0 \\ 0, & \text{otherwise} \end{cases}$ and hence find its mean and variance. (N/D 2012)

• Problems on distributions

1. If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8. What is the probability that he will finally pass the test

   (1) On the fourth trial and

   (2) In less than 4 trials? (A/M 2010)

2. The marks obtained by a number of students in a certain subject are assumed to be normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group, what is the probability that two of them will have marks over 70?(A/M 2010)

3. The marks obtained by a number of students in a certain subject are assumed to be normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this set, what is the probability that exactly 2of them will have marks over 70?(A/M 2011)

4. Assume that the reduction of a person's oxygen consumption during a period of Transcendental Meditation (T.M) is a continuous random variable X normally distributed with mean 37.6 cc/mm and S.D 4.6 cc/min. Determine the probability that during a period of T.M. a person's oxygen consumption will be reduced by

(1) at least 44.5 cc/min

(2) at most 35.0 cc/min

(3) anywhere from 30.0 to 40.0 cc/mm. (N/D 2012)

5. Let X and Y be independent normal variates with mean 45 and 44 and standard deviation 2 and 1.5 respectively. What is the probability that randomly chosen values of X and Y differ by 1.5 or more?(N/D 2011)

6. Given that X is distributed normally, if $P(X < 45) = 0.31$ and $P(X > 64) = 0.08$,

find the mean and standard deviation of the distribution. (M/J 2012)

7. If X and Y are independent random variables following $N(8, 2)$ and $N\left(12, 4\sqrt{3}\right)$ respectively, find the value of λ such that $P[2X - Y \le 2\lambda] = P[X + 2Y \ge \lambda]$.

(N/D 2010)

8. The time in hours required to repair a machine is exponentially distributed with parameter $\lambda = 1/2$.

(1) What is the probability that the repair time exceeds 2 hours?

(2) What is the conditional probability that a repair takes atleast 10 hours given that its duration exceeds 9 hours? (M/J 2012)

- **Function of random variable**

1. If X is uniformly distributed in $(-1,1)$, then find the probability density function of $Y = \sin\frac{\pi X}{2}$. (N/D 2010)

2. If X is a uniform random variable in the interval @, find the probability density function $Y = X$ and $E[Y]$.(N/D 2011)

3. The random variable X has exponential distribution with $f(x) = \begin{cases} e^{-x}, & 0 < x < \infty \\ 0, & \text{otherwise} \end{cases}$.

Find the density function of the variable given by

(1) $Y = 3X + 5$

(2) $Y = X^2$ (N/D 2012)

Unit – II (Two Dimensional Random Variables form)

• Joint distributions – Marginal & Conditional

1. The joint p.d.f of two dimensional random variable (X,Y) is given by $f(x,y) = \frac{8}{9}xy$, $0 \le x \le y \le 2$ and $f(x,y) = 0$, otherwise. Find the densities of X and Y, and the conditional densities $f(x/y)$ and $f(y/x)$. (A/M 2010)

2. The joint probability density function of random variable X and Y is given by $f(x,y) = \begin{cases} \frac{8xy}{9}, & 1 \le x \le y \le 2 \\ 0, & \text{otherwise} \end{cases}$. Find the conditional density functions of X and Y. (N/D 2011)

3. The joint pdf of a two-dimensional random variable (X,Y) is given by $f(x,y) = xy^2 + \frac{x^2}{8}, 0 \le x \le 2, 0 \le y \le 1$. Compute $P(Y < 1/2)$, $P(X > 1 / Y < 1/2)$ and $P(X + Y \le 1)$. (N/D 2012)

4. Find the bivariate probability distribution of (X,Y) given below:

Y123456
X
0001/32 2/32 2/32 3/32
1 1/16 1/16 1/8 1/8 1/8 1/8
2 1/32 1/32 1/64 1/6402/64

Find the marginal distributions, conditional distribution of X given Y = 1 and conditional distribution of Y given X = 0.(A/M 2010)

• Covariance, Correlation and Regression

1. Find the covariance of X and Y, if the random variable (X,Y) has the joint p.d.f $f(x,y) = x + y, 0 \le x \le 1, 0 \le y \le 1$ and $f(x,y) = 0$, otherwise. (A/M 2010)

2. The joint probability density function of random variable (X, Y) is given by $f(x,y) = Kxye^{-(x^2+y^2)}, x > 0, y > 0$. Find the value of K and $Cov(X, Y)$. Are X and Y independent? (M/J 2012)

3. The joint probability density function of the two dimensional random variable (X, Y) is $f(x, y) = \begin{cases} 2 - x - y, & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$. Find the correlation coefficient between X and Y. (N/D 2011)

4. Two random variables X and Y have the joint probability density function given by

$$f_{XY}(x, y) = \begin{cases} k(1 - x^2 y), & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}.$$

(1) Find the value of 'k'

(2) Obtain the marginal probability density functions of X and Y.

(3) Also find the correlation coefficient between X and Y.

(N/D 2010)

5. If X and Y are uncorrelated random variables with variances 16 and 9. Find the correlation co-efficient between $X + Y$ and $X - Y$. (M/J 2012)

6. If the independent random variables X and Y have the variances 36 and 16 respectively, find the correlation coefficient between $(X + Y)$ and $(X - Y)$.

(N/D 2012)

7. The regression equation of X on Y is $3Y - 5X + 108 = 0$. If the mean value of Y is 44 and the variance of X is 9/16th of the variance of Y. Find the mean value of X and the correlation coefficient. (A/M 2011)

• Transformation of the random variables

1. If X and Y are independent random variables with density function $f_X(x) = \begin{cases} 1, & 1 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$ and $f_Y(y) = \begin{cases} \frac{y}{6}, & 2 \le y \le 4 \\ 0, & \text{otherwise} \end{cases}$, find the density function of $Z = XY$. (A/M 2011)

2. If X and Y are independent with a common PDF (exponential): $f(x) = \begin{cases} e^{-x}, & x \ge 0 \\ 0, & x < 0 \end{cases}$ and $f(y) = \begin{cases} e^{-y}, & y \ge 0 \\ 0, & y < 0 \end{cases}$. Find the PDF for $X - Y$. (N/D 2011)

3. If X and Y are independent random variables with probability density functions $f_X(x) = 4e^{-4x}, x \ge 0$; $f_Y(y) = 2e^{-2y}, y \ge 0$ respectively.

(i) Find the density function of $U = \frac{X}{X+Y}$, $V = X+Y$

(ii) Are U and V independent?

(iii) What is $P(U > 0.5)$?

4. Let (X, Y) be a two dimensional random variable and the probability density function be given by $f(x, y) = x + y,\ 0 \le x, y \le 1$. Find the p.d.f of $U = XY$. (M/J 2012)

5. If X and Y are independent continuous random variables, show that the pdf of $U = X + Y$ is given by $h(u) = \int_{-\infty}^{\infty} f_X(v) f_Y(u - v)dv$. (N/D 2010)

• Central Limit Theorem

1. A sample of size 100 is taken from a population whose mean is 60 and variance is 400. Using Central Limit Theorem, find the probability with which the mean of the sample will not differ from 60 by more than 4.(A/M 2010)

2. The life time of a particular variety of electric bulb may be considered as a random variable with mean 1200 hours and standard deviation 250 hours. Using central limit theorem, find the probability that the average life time of 60 bulbs exceeds 1250 hours. (A/M 2011)

3. Let $X_1, X_2, X_3, \ldots X_n$ be Poisson variates with parameter $\lambda = 2$ and $S_n = X_1 + X_2 + X_3 + \ldots + X_n$ where $n = 75$. Find $p\left[120 \le S_n \le 160\right]$ using central limit theorem. (M/J 2012)

4. If $X_1, X_2, X_3, \ldots X_n$ are uniform variates with mean $= 2.5$ and variance $= 3/4$, use CLT to estimate $p(108 \le S_n \le 12.6)$ where $S_n = X_1 + X_2 + X_3 + \ldots + X_n$, $n = 48$.

(N/D 2011)

5. If V_i, $i = 1, 2, 3 \ldots 20$ are independent noise voltages received in an adder and V is the sum of the voltages received, find the probability that the total incoming voltage V exceeds 105, using the central limit theorem. Assume that each of the random variables V_i is uniformly distributed over (0,10). (N/D 2010)

Unit – III (Classification of Random Processes)

• Verification of SSS and WSS process

1. Examine whether the random process $\{X(t)\} = A\cos(\omega t + \theta)$ is a wide sense stationary if A and ω are constants and θ is uniformly distributed random variable in $(0,2\pi)$.(A/M 2010),(N/D 2011)

2. A random process $X(t)$ defined by $X(t) = A\cos t + B\sin t$, $-\infty < t < \infty$, where A and B are independent random variables each of which takes a value -2 with probability $1/3$ and a value 1 with probability $2/3$. Show that $X(t)$ is wide – sense stationary. (A/M 2011)

3. The process $\{X(t)\}$ whose probability distribution under certain condition is given by

$$P\{X(t)=n\} = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1, 2\ldots \\ \dfrac{at}{1+at}, & n=0 \end{cases}$$

. Find the mean and variance of the process. Is the process first-order stationary? (N/D 2010),(N/D 2011),(N/D 2012)

4. If $\{X(t)\}$ is a WSS process with autocorrelation $R(\tau) = Ae^{-\alpha|\tau|}$, determine the second order moment of the RV $\{X(8) - X(5)\}$. (M/J 2012)

• Ergodic Processes, Mean ergodic and Correlation ergodic

1. The random binary transmission process $\{X(t)\}$ is a wide sense process with zero mean and autocorrelation function $R(\tau) = 1 - \dfrac{|\tau|}{T}$, where T is a constant. Find the mean and variance of the time average of $\{X(t)\}$ over (0, T). Is $\{X(t)\}$ mean – ergodic?

(A/M 2010)

2. A random process has sample functions of the form $X(t) = A\cos(\omega t + \theta)$, where ω is constant, A is a random variable with mean zero and variance one and θ is a random variable that is uniformly distributed between 0 and 2π. Assume that the random variables A and θ are independent. Is $X(t)$ is a mean – ergodic process?

(A/M 2011)

3. If the WSS process $\{X(t)\}$ is given by $X(t) = 10\cos(100t + \theta)$, where θ is uniformly distributed over $(-\pi, \pi)$, prove that $\{X(t)\}$ is correlation ergodic.

(N/D 2010),(M/J 2012),(N/D 2012)

• Problems on Markov Chain

1. The transition probability matrix of a Markov chain $\{X(t)\}$, $n = 1, 2, 3, \ldots$ having three states $1, 2, 3$ is $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$, and the initial distribution is $P^{(0)} = [0.7\ 0.2\ 0.1]$, Find $P(X_2 = 3)$ and $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$.

(A/M 2010)

• Poisson process

1. If the process $\{X(t); t \geq 0\}$ is a Poisson process with parameter λ, obtain $P[X(t) = n]$. Is the process first order stationary? (N/D 2010),(N/D 2012)
2. State the postulates of a Poisson process and derive the probability distribution. Also prove that the sum of two independent Poisson processes is a Poisson process. (N/D 2011)
3. If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is
   (1) more that 1 minute
   (2) between 1 minute and 2 minute and
   (3) 4 min. or less.(M/J 2012)
4. Assume that the number of messages input to a communication channel in an interval of duration t seconds, is a Poisson process with mean $\lambda = 0.3$. Compute
   (1) The probability that exactly 3 messages will arrive during 10 second interval
   (2) The probability that the number of message arrivals in an interval of duration 5 seconds is between 3 and 7.(A/M 2010)
5. Prove that the interval between two successive occurrences of a Poisson process with parameter λ has an exponential distribution with mean $\frac{1}{\lambda}$. (A/M 2011)

• Normal (Gaussian) & Random telegraph Process

1. If $\{X(t)\}$ is a Gaussian process with $\mu(t) = 10$ and $C(t_1, t_2) = 16e^{-|t_1 - t_2|}$, find the probability that

   (1) $X(10) \le 8$

   (2) $|X(10) - X(6)| \le 4$ (A/M 2011)

2. Suppose that $X(t)$ is a Gaussian process with $\mu_x = 2$, $R_{xx}(\tau) = 5e^{-0.2|\tau|}$. Find the probability that $X(4) \le 1$. (M/J 2012)

3. Prove that a random telegraph signal process $Y(t) = \alpha X(t)$ is a Wide Sense Stationary Process when α is a random variable which is independent of $X(t)$, assume value -1 and $+1$ with equal probability and $R_{XX}(t_1, t_2) = e^{-2\lambda|t_1 - t_2|}$. (N/D 2010),(N/D 2012)

Unit – IV (Correlation and Spectral densities)

• Auto Correlation from the given process

1. Find the autocorrelation function of the periodic time function of the period time function $\{X(t)\} = A \sin \omega t$. (A/M 2010)

• Relationship between $R_{XX}(\tau)$ and $S_{XX}(\omega)$

1. The autocorrelation function of the random binary transmission $\{X(t)\}$ is given by $R(\tau) = 1 - \frac{|\tau|}{T}$ for $|\tau| < T$ and $R(\tau) = 0$ for $|\tau| < T$. Find the power spectrum of the process $\{X(t)\}$. (A/M 2010)

2. Find the power spectral density of the random process whose auto correlation function is $R(\tau) = \begin{cases} 1 - |\tau|, & \text{for } |\tau| \le 1 \\ 0, & \text{elsewhere} \end{cases}$. (N/D 2010),(N/D 2012)

3. Find the power spectral density function whose autocorrelation function is given by $R_{XX}(\tau) = \frac{A^2}{2}\cos(\omega_0\tau)$. (M/J 2012)

4. The autocorrelation function of a random process is given by

$$R(\tau) = \begin{cases} \lambda^2; & |\tau| > \varepsilon \\ \lambda^2 + \dfrac{\lambda}{\varepsilon}\left(1 - \dfrac{|\tau|}{\varepsilon}\right); & |\tau| \le \varepsilon \end{cases}$$

. Find the power spectral density of the process. (N/D 2011)

5. The Auto correlation function of a WSS process is given by $R(\tau) = \alpha^2 e^{-2\lambda|\tau|}$ determine the power spectral density of the process. (A/M 2011)

6. Find the power spectral density of a WSS process $X(t)$ which has an autocorrelation $R_{xx}(\tau) = A_0\left[1 - |\tau|/T\right], -T \le t \le T.$ (N/D 2012)

7. Find the autocorrelation function of the process $\{X(t)\}$ for which the power spectral density is given by $S_{XX}(\omega) = 1 + \omega^2$ for $|\omega| < 1$ and $S_{XX}(\omega) = 0$ for $|\omega| > 1$.(A/M 2010)

8. The power spectral density function of a zero mean WSS process $X(t)$ is given by

$$S(\omega) = \begin{cases} 1, & |\omega| < \omega_0 \\ 0, & \text{otherwise} \end{cases}$$

. Find $R(\tau)$ and show that $X(t)$ and $X\left(t + \dfrac{\pi}{\omega_0}\right)$ are uncorrelated. (A/M 2011)

• Relationship between $R_{XY}(\tau)$ and $S_{XY}(\omega)$

1. The cross-correlation function of two processes $X(t)$ and $Y(t)$ is given by

$$R_{XY}(t, t+\tau) = \frac{AB}{2}\left\{\sin(\omega_0\tau) + \cos\omega_0(2t+\tau)\right\}$$

where A, B and ω_0 are constants. Find the cross-power spectrum $S_{XY}(\omega)$.(M/J 2012)

2. The cross – power spectrum of real random processes $\{X(t)\}$ and $\{Y(t)\}$ is given by

$$S_{xy}(\omega) = \begin{cases} a + bj\omega, & \text{for } |\omega| < 1 \\ 0, & \text{elsewhere} \end{cases}$$

. Find the cross correlation function.

(N/D 2010),(A/M 2011),(N/D 2011)

• Properties, Theorem and Special problems

1. State and prove Weiner – Khintchine Theorem.
(N/D 2010),(A/M 2011),(N/D 2011),(N/D2012)

2. If $\{X(t)\}$ and $\{Y(t)\}$ are two random processes with auto correlation function $R_{XX}(\tau)$ and $R_{YY}(\tau)$ respectively then prove that $R_{XY}(\tau) \le R_{XX}(0)\, R_{YY}(0)$.

Establish any two properties of auto correlation function $R_{XX}(\tau)$.(N/D 2010),(N/D2012)

3. Given the power spectral density of a continuous process as $S_{XX}(\omega) = \frac{\omega^2 + 9}{\omega^4 + 5\omega^2 + 4}$

Find the mean square value of the process.(N/D 2011)

4. A stationary random process $X(t)$ with mean 2 has the auto correlation function $R_{XX}(\tau) = 4 + e^{\frac{-\tau}{10}}$. Find the mean and variance of $Y = \int_0^1 X(t)\, dt$. (M/J 2012)

5. $\{X(t)\}$ and $\{Y(t)\}$ are zero mean and stochastically independent random processes having autocorrelation functions $R_{XX}(\tau) = e^{-\tau}$ and $R_{YY}(\tau) = \cos 2\pi\tau$ respectively.

Find

(1) The autocorrelation function of $W(t) = X(t) + Y(t)$ and $Z(t) = X(t) - Y(t)$

(2) The cross correlation function of $W(t)$ and $Z(t)$. (A/M 2010)

6. Let $X(t)$ and $Y(t)$ be both zero-mean and WSS random processes Consider the random process $Z(t)$ defined by $Z(t) = X(t) + Y(t)$. Find

(1) The Auto correlation function and the power spectrum of $Z(t)$ if $X(t)$ and $Y(t)$ are jointly WSS.

(2) The power spectrum of $Z(t)$ if $X(t)$ and $Y(t)$ are orthogonal.

(M/J 2012)

Unit – V (Linear systems with Random inputs)

• Input and Output process

1. If the input to a time invariant, stable, linear system is a WSS process, prove that the output will also be a WSS process.(N/D 2011)

2. Show that if the input $\{X(t)\}$ is a WSS process for a linear system then output $\{Y(t)\}$ is a WSS process. Also find $R_{XY}(\tau)$. (N/D 2010),(N/D 2012)

3. For a input – output linear system $(X(t), h(t), Y(t))$, derive the cross correlation function $R_{XY}(\tau)$ and the output autocorrelation function $R_{YY}(\tau)$. (N/D 2011)

4. Consider a system with transfer function $\frac{1}{1+j\omega}$. An input signal with autocorrelation function $m\delta(\tau) + m^2$ is fed as input to the system. Find the mean and mean-square value of the output. (A/M 2011),(M/J 2012)

5. If $\{X(t)\}$ is a WSS process and if $Y(t) = \int_{-\infty}^{\infty} h(\xi)X(t-\xi)d\xi$ then prove that

   (1) $R_{XY}(\tau) = R_{XX}(\tau) * h(\tau)$ where * stands for convolution.

   (2) $S_{XY}(\omega) = S_{XX}(\omega)H^*(\omega)$. (M/J 2012)

6. Assume a random process $X(t)$ is given as input to a system with transfer function $H(\omega) = 1$ for $-\omega_0 < \omega < \omega_0$. If the autocorrelation function of the input process is $\frac{N_0}{2}\delta(t)$, find the autocorrelation function of the output process. (A/M 2010)

7. If $X(t)$ is the input voltage to a circuit and $Y(t)$ is the output voltage. $\{X(t)\}$ is a stationary random process with $\mu_X = 0$ and $R_{XX}(\tau) = e^{-2|\tau|}$. Find the mean μ_Y and power spectrum $S_{YY}(\omega)$ of the output if the system transfer function is given by $H(\omega) = \frac{1}{\omega + 2i}$. (N/D 2010),(N/D 2012)

• Input and Output process with impulse response

1. A system has an impulse response $h(t) = e^{-\beta t}U(t)$, find the power spectral density of the output $Y(t)$ corresponding to the input $X(t)$. (N/D 2010),(N/D 2012)

2. A stationary random process $X(t)$ having the autocorrelation function $R_{XX}(\tau) = A\delta(\tau)$ is applied to a linear system at time $t = 0$ where $f(\tau)$ represent the impulse function. The linear system has the impulse response of $h(t) = e^{-bt}u(t)$ where $u(t)$ represents the unit step function. Find $R_{YY}(\tau)$. Also find the mean and variance of $Y(t)$. (A/M 2011),(M/J 2012)

3. A wide sense stationary random process $\{X(t)\}$ with autocorrelation $R_{XX}(\tau) = e^{-a\tau}$ where A and a are real positive constants, is applied to the input of an Linear transmission input system with impulse response $h(t) = e^{-bt}u(t)$ where b is a real positive constant. Find the autocorrelation of the output $Y(t)$ of the system.(A/M 2010)

4. A linear system is described by the impulse response $h(t) = \frac{1}{RC}e^{\frac{-t}{RC}}u(t)$. Assume an input process whose Auto correlation function is $B\delta(\tau)$. Find the mean and Auto correlation function of the output process. (A/M 2011)

5. Let $X(t)$ be a WSS process which is the input to a linear time invariant system with unit impulse $h(t)$ and output $Y(t)$, then prove that $S_{yy}(\omega) = |H(\omega)|^2 S_{xx}(\omega)$.

(N/D 2011)

• Band Limited White Noise

1. If $Y(t) = A\cos(\omega_0 t + \theta) + N(t)$, where A is a constant, θ is a random variable with a uniform distribution in $(-\pi, \pi)$ and $\{N(t)\}$ is a band-limited Gaussian white noise with power spectral density $S_{NN}(\omega) = \begin{cases} \frac{N_0}{2}, & \text{for } |\omega - \omega_0| < \omega_B \\ 0, & \text{elsewhere} \end{cases}$. Find the power spectral density $\{Y(t)\}$. Assume that $\{N(t)\}$ and θ are independent.

(N/D 2010),(N/D 2012)

2. If $Y(t) = A\cos(\omega t + \theta) + N(t)$, where A is a constant, θ is a random variable with a uniform distribution in $(-\pi, \pi)$ and $\{N(t)\}$ is a band limited Gaussian white noise with a power spectral density $S_{NN}(\omega) = \frac{N_0}{2}$ for $|\omega - \omega_0| < \omega_B$ and $S_{NN}(\omega) = 0$, elsewhere. Find the power spectral density of $Y(t)$, assuming that $N(t)$ and θ are independent. (A/M 2010)

3. If $\{N(t)\}$ is a band limited white noise centered at a carrier frequency ω_0 such that $S_{NN}(\omega) = \begin{cases} \frac{N_0}{2}, & \text{for } |\omega - \omega_0| < \omega_B \\ 0, & \text{elsewhere} \end{cases}$. Find the autocorrelation of $\{N(t)\}$. (A/M 2011),(M/J 2012)

4. If $\{X(t)\}$ is a band limited process such that $S_{XX}(\omega) = 0$ when $|\omega| > \sigma$, prove that $2[R_{XX}(0) - R_{XX}(\tau)] \le \sigma^2\tau^2 R_{XX}(0)$. (A/M 2010)

5. A white Gaussian noise $X(t)$ with zero mean and spectral density $\frac{N_0}{2}$ is applied to a low-pass RC filter shown in the figure.

Determine the autocorrelation of the output $Y(t)$. (N/D 2011)

----All the Best----