SUBJECT NAME : Numerical Methods

SUBJECT CODE : MA2264

MATERIAL NAME : University Questions

MATERIAL CODE : SKMA1005

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Name of the Student: Branch:

Unit – I (Solution of Equations and Eigenvalue Problems)

• Fixed point iteration

1) Solve $y = 3e^{x} - 3x = 0$ by the method of fixed point iteration. (M/J 2012)

2) Find a real root of the equation $x^3 + x^2 - 1 = 0$ by iteration method. (M/J 2012)

3) Find a positive root of the equation $\cos x - 3x + 1 = 0$ by using iteration method. (M/J 2013)

• Newton's method (or) Newton Raphson method

1) Solve for a positive root of the equation $x^4 - x - 10 = 0$ using Newton – Raphson method.(A/M 2010)

2) Solve $x \sin x + \cos x = 0$ using Newton-Raphson method. (M/J 2012)

3) Find an iterative formula to find the reciprocal of a given number N and hence find the value of $\frac{1}{19}$. (N/D 2011)

4) Find theNewton's iterative formula to calculate the reciprocal N and hence find the value of $\frac{1}{23}$. (N/D 2012)

5) Find an iterative formula to find N , where N is a positive number and hence find 5. (N/D 2012)

• Solution of linear system by Gaussian elimination method

1) Apply Gauss elimination method to find the solution of the following system: $2x+3y-z=5,\ 4x+4y-3z=3,\ 2x-3y+2z=2$. (N/D 2012)

• Solution of linear system by Gaussian-Jordon method

1) Apply Gauss-Jordan method to find the solution of the following system:

$10x+y+z=12;\ 2x+10y+z=13;\ x+y+5z=7$. (N/D 2011)

• Solution of linear system by Gaussian-Seidel method

1) Apply Gauss-Seidal method to solve the equations $20x+y-2z=17;\ 3x+20y-z=-18;\ 2x-3y+20z=25$. (M/J 2012)

2) Solve, by Gauss-Seidel method, the following system: $28x+4y-z=32;\ x+3y+10z=24;\ 2x+17y+4z=35$. (N/D 2011)

3) Use Gauss – Seidal iterative method to obtain the solution of the equations: $9x-y+2z=9;\ x+10y-2z=15;\ 2x-2y-13z=-17$. (A/M 2010)

4) Solve the following system of equations using Gauss-Seidel method: $10x+2y+z=9;\ x+10y-z=-22;\ -2x+3y+10z=22$. (N/D 2012)

5) Solve, by Gauss-Seidel method, the following system: $8x-3y+2z=20;\ 4x+11y-z=33;\ 6x+3y+12z=35$. (N/D 2012)

6) Solve, by Gauss-Seidel method, the equations $27x+6y-z=85;\ 6x+15y+2z=72;\ x+y+54z=110$. (M/J 2013)

• Inverse of a matrix by Gauss Jordon method

1) Using Gauss Jordan method, find the inverse of the matrix $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$. (M/J 2012)

2) Find the inverse of the matrix by Gauss – Jordan method: $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$. (A/M 2010)

3) Using Gauss Jordan method, find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{bmatrix}$.(N/D 2012)

4) Find, by Gauss-Jordan method, the inverse of the matrix $A = \begin{bmatrix} 2 & 4 & 1 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{bmatrix}$.(M/J 2013)

• Eigen value of matrix by power and Jacobi method

1) Determine the largest eigen value and the corresponding eigen vector of the matrix

$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. (M/J 2012)

2) Find the largest eigenvalue of $\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$, by using Power method.

(A/M 2010),(N/D 2011),(M/J 2012),(N/D 2012)

3) Find all the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ using Jacobi method. (N/D 2012)

4) Using Jacobi method find the all eigen values and their corresponding eigen vectors of the matrix $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}$. (M/J 2013)

Unit – II (Interpolation and Approximation)

• Lagrange Polynomials and Divided differences

1) Using Lagrange's interpolation, calculate the profit in the year 2000 from the following data:(M/J 2012)

Year:	1997	1999	2001	2002
Profit Lakhs Rs.	43	65	159	248

2) Using Lagrange's interpolation formula find the value of y when $x = 10$, if the values of x and y are given below: (M/J 2012)

x: 5 6 9 11
y : 12 13 14 16

3) Apply Lagrange's formula, to find $y\,(27)$ to the data given below. (M/J 2013)

x : 14 17 31 35
y : 68.8 64 44 39.1

4) Use Lagrange's formula to find a polynomial which takes the values $f(0) = -12, f(1) = 0, f(3) = 6$ and $f(4) = 12$. Hence find $f(2)$. (A/M 2010)

5) Use Lagrange's method to find $log_{10} 656$, given that $log_{10} 654 = 2.8156, log_{10} 658 = 2.8182, log_{10} 659 = 2.8189$ and (N/D 2012) $log_{10} 661 = 2.8202$.

6) Determine $f(x)$ as a polynomial in x for the following data, using Newton's divided difference formulae. Also find $f(2)$.(N/D 2011)

x: –4–1 0 2 5
$f(x)$: 1245 33 5 9 1335

7) Find the function $f(x)$ from the following table using Newton's divided difference formula:(A/M 2010)

0 1 2 4 57x:
$f(x)$: 0 0 –12 0 600 7308

8) Using Newton's divided differences formula determine $f(3)$ from the data:

x: 0 1 2 4 5 (M/J 2012)
$f(x)$: 1 14 15 5 6

9) Use Newton's divided difference formula to find $f(x)$ form the following data.

x: 1 2 7 8 (M/J 2013)
y: 1 5 5 4

10) Using Newton's divided difference formula, find $f(x)$ from the following data and hence find $f(4)$. (N/D 2012)

x	0	1	2	5
$f(x)$	2	3	12	147

• Interpolating with a cubic spline

1) The following values of x and y are given:

x: 1 2 3 4
y : 1 2 5 11

Find the cubic splines and evaluate $y(1.5)$ and $y'(3)$. (M/J 2012)

2) From the following table:

x: 1 2 3
y : - 8 -1 18

Compute $y(1.5)$ and $y'(1)$ using cubic sphere. (M/J 2012),(M/J 2013)

3) Obtain the cubic spline for the following data to find $y(0.5)$. (N/D 2012)

x : -1 0 1 2
y : -1 1 3 35

4) If $f(0) = 1$, $f(1) = 2$, $f(2) = 33$ and $f(3) = 244$. find a cubic spline approximation,

assuming M(0) = M(3) = 0. Also, find $f(2.5)$. (A/M 2010)

• Newton's forward and backward difference formulas

1) Find the cubic polynomial which takes the following values: (M/J 2012),(M/J 2013)

x: 0 1 2 3
$f(x)$: 1 2 1 10

2) Find the value of **y** when **x** using Newton's interpolation formula from the following table:(N/D 2012)

x 4 6 8 10
y 1 3 8 16

3) Find the value of y at $x = 21$ and $x = 28$ from the data given below (N/D2012)

x: 20232629
y : 0.3420 0.3907 0.4384 0.4848

4) The population of a town is as follows:

x Year:1941 1951 1961 1971 1981 1991
y Population in thousands:202429364651

Estimate the population increase during the period 1946 to 1976. (N/D 2011)

5) Given the following table, find the number of students whose weight is between 60 and 70 lbs:(A/M 2010),(M/J 2012)

Weight (in lbs) x: 0 – 40 40 – 60 60 – 80 80 – 100 100 – 120
No. of students:2501201007050

Unit – III (Numerical Differentiation and Integration)

• Differentiation using interpolation formulae

1) A slider in a machine moves along a fixed straight rod. Its distance x cm along the rod is given below for various values of the time $'t'$ seconds. Find the velocity of the slider when $t = 1.1$ second.(M/J 2012)

1.01.11.21.31.41.51.6t:
x : 7.989 8.403 8.781 9.129 9.451 9.750 10.031

2) Find $f'(x)$ at $x = 1.5$ and $x = 4.0$ from the following data using Newton's formulae for differentiation. (N/D2012)

x: 1.5 2.02.53.03.54.0
$y = f(x)$: 3.375 7.0 13.625 24.0 38.875 59.0

3) Find the first three derivatives of $f(x)$ at $x = 1.5$ by Newton's forward interpolation formla to the data given below. (M/J 2013)

x: 1.5 2.02.53.03.54.0
$y = f(x)$: 3.375 7.0 13.625 24.0 38.875 59.0

4) Given the following data, find $y'(6)$ and the maximum value of **y** (if it exists)

x: 0 2 3479 (A/M 2010)
y : 4 26 58 112 466 922

5) Find the first two derivatives of $x^{1/3}$ at $x = 50$ and $x = 56$, for the given table: (N/D 2011)

x: 50515253545556
$y = x^{1/3}$ 3.6840 3.7084 3.7325 3.7563 3.7798 3.8030 3.8259

6) Find the first and second derivative of the function tabulated below at $x = 0.6$. (N/D 2012)

x: 0.40.50.60.70.8
y : 1.5836 1.7974 2.0442 2.3275 2.6511

7) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 51$, from the following data: (M/J 2012)

x: 5060708090
y : 19.96 36.65 58.81 77.21 94.61

• Numerical integration by trapezoidal, Simpson's 1/3, 3/8 rules & Romberg's method

1) Use Romberg's method to compute $\int_0^1 \frac{dx}{1+x^2}$ correct to 4 decimal places. Also evaluate the same integral using tree-point Gaussian quadrature formula. Comment on the obtained values by comparing with the exact value of the integral which is equal to $\pi/4$. (M/J 2012)

2) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Romberg's method correct to 4 decimal places. Hence deduce an approximate value for π . (M/J 2012)

3) Using Romberg's integration to evaluate $\int_0^1 \frac{dx}{1+x^2}$. (A/M 2010)

4) Using Trapezoidal rule, evaluate $\int_{-1}^1 \frac{dx}{1+x^2}$ by taking eight equal intervals. (M/J 2013)

5) Compute $\int_0^{\pi/2} sin\ x\ dx$ using Simpson's 3/8 rule. (N/D 2012)

6) Evaluate $I = \int_0^{2\pi} sin\ x\ dx$ by dividing the range into ten equal parts, using

(i) Trapezoidal rule (N/D2012)

(ii) Simpson's one-third rule, Verify your answer with actual integration.

7) Evaluate $I = \int_0^6 \frac{1}{1+x}\,dx$ by using (i) Direct integration (ii) Trapezoidal rule (iii) Simpson's one-third rule (iv) Simpson's three-eighth rule. (N/D 2011)

8) The velocity υ of a particle at a distance S from a point on its path is given by the table below:

S (meter)	0	10	20	30	40	50	60
υ (m */ sec*)	47	58	64	65	61	52	38

Estimate the time taken to travel 60 meters by Simpson's 1/3rd rule and Simpson's 3/8th rule.(A/M 2010)

• Two and Three point Gaussian Quadrature formulae

1) Use Gaussian three-point formula and evaluate $\int_1^5 \frac{dx}{x}$. (M/J 2012)

2) Evaluate $\int_0^2 \frac{x^2 + 2x + 1}{1 + (x+1)^2}\,dx$ by Gaussian three point formula. (M/J 2013)

• Double integrals using Trapezoidal and Simpsons's rules

1) Using Trapezoidal rule, evaluate $\int_1^2\int_1^2 \frac{dxdy}{x^2+y^2}$ numerically with $h = 0.2$ along x -direction and $k = 0.25$ along y -direction. (M/J 2012)

2) Evaluate $\int_1^{1.2}\int_1^{1.4} \frac{dxdy}{x+y}$ by trapezoidal formula by taking $h = k = 0.1$. (A/M 2010)

3) Evaluate $\int_0^2\int_0^1 4xy\,dxdy$ using Simpson's rule by taking $h = \frac{1}{4}$ and $k = \frac{1}{2}$.(N/D 2012)

4) Evaluate $\int_0^{1/2}\int_0^{1/2} \frac{\sin(xy)}{1+xy}\,dxdy$ using Simpson's rule with $h = k = \frac{1}{4}$. (M/J 2012)

5) Evaluate $\int_1^{1.2}\int_2^{2.4} \frac{1}{xy}\,dxdy$ using Simpon's one-third rule. (M/J 2013)

Unit – IV (Initial Value Problems for Ordinary Differential Equations)

• Taylor series method

1) Use Tailor series method to find $y\ (0.1)$ and $y\ (0.2)$ given that $\frac{dy}{dx} = 3e^{x} + 2y$, $y\ (0) = 0$, correct to 4 decimal accuracy. (A/M 2010)

2) Using Taylor series method solve $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$ at $x = 0.1, 0.2, 0.3$. Also compare the values with exact solution. (M/J 2012)

3) Using Taylor series method to find $y\ (0.1)$ if $y' = x^2 + y^2$, $y(0) = 1$. (M/J 20113)

• Euler method for first order equation

1) Using Modified Euler's method, find $y\ (4.1)$ and $y\ (4.2)$ if

$5x \frac{dy}{dx} + y^2 - 2 = 0;\ y(4) = 1$. (N/D 2012)

2) Solve $y' = \frac{y - x}{y + x}$, $y(0) = 1$ at $x = 0.1$ by taking $h = 0.02$ by using Euler's method. (M/J 2013)

• Fourth order Runge – Kutta method

1) Use Runge – Kutta method of fourth order to find $y\ (0.2)$, given, $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y\ (0) = 1$, taking $h = 0.2$.(A/M 2010)

2) Consider the second order initial value problem $y'' - 2y' + 2y = e^{2t} \sin t$ with $y\ (0) = -0.4$ and $y\ '(0) = -0.6$ using Fourth order Runge Kutta algorithm, find $y\ (0.2)$. (M/J 2012)

3) Solve $y'' = xy' - y$ given $y\ (0) = 3,\ y\ '(0) = 0$ to find the value of $y\ (0.1)$, using RK-method of order 4. (M/J 2012)

4) Given $y'' + xy' + y = 0,\ y\ (0) = 1,\ y\ '(0) = 0$. Find the value of $y\ (0.1)$ by using Runge-Kutta method of fourth order. (N/D 2011)

5) Given $y'' - x(y')^2 + y^2 = 0$, $y(0) = 1$, $y'(0) = 0$. Find the value of $y(0.2)$ by using Runge-Kutta method of fourth order, by taking $h = 0.2$. (N/D 2012)

6) Using Runge-Kutta method find $y(0.2)$ if $y'' = xy'^2 - y^2$, $y(0) = 1$, $y'(0) = 0, h = 0.2$. (M/J 2013)

7) Solve for $y(0.1)$ and $z(0.1)$ from the simultaneous differential equations $\frac{dy}{dx} = 2y + z; \frac{dz}{dx} = y - 3z; y(0) = 0, z(0) = 0.5$ using Runge-Kutta method of the fourth order. (N/D 2012)

• Milne's and Adam's predictor and corrector methods

1) Given that $y'' + xy' + y = 0, y(0) = 1, y'(0) = 0$ obtain y for $x = 0.1, 0.2$ and 0.3 by Taylor's series method and find the solution for $y(0.4)$ by Milne's method.(M/J 2012)

2) Given that $\frac{dy}{dx} = \frac{1}{2}(1 + x^2)y^2; y(0) = 1; y(0.1) = 1.06; y(0.2) = 1.12$ and $y(0.3) = 1.21$, evaluate $y(0.4)$ and $y(0.5)$ by Milne's predictor corrector method. (N/D 2011)

3) Given that $\frac{dy}{dx} = 1 + y^2; y(0.6) = 0.6841, y(0.4) = 0.4228, y(0.2) = 0.2027,$ $y(0) = 0$, find $y(-0.2)$ using Milne's method.(N/D 2012)

4) Use Milne's predictor – corrector formula to find $y(0.4)$, given $\frac{dy}{dx} = \frac{(1 + x^2)y^2}{2}$, $y(0) = 1, y(0.1) = 1.06, y(0.2) = 1.12$ and $y(0.3) = 1.21$.(A/M 2010)

5) Given $y' = \frac{1}{x + y}$, $y(0) = 2, y(0.2) = 2.0933, y(0.4) = 2.1755$, $y(0.6) = 2.2493$ find $y(0.8)$ using Milne's method. (M/J 2012)

6) Given $\frac{dy}{dx} = xy + y^2, y(0) = 1, y(0.1) = 1.1169, y(0.2) = 1.2774, y(0.3) = 1.5041$. Use Adam's method to estimate $y(0.4)$.(A/M 2010)

7) Using Adams method find $y(1.4)$ given $y' = x^2(1 + y), y(1) = 1, y(1.1) = 1.233,$ $y(1.2) = 1.548$ and $y(1.3) = 1.979$.(M/J 2012)

8) Given that $y' = y - x^2$; $y(0) = 1$; $y(0.2) = 1.1218$; $y(0.4) = 1.4682$ and $y(0.6) = 1.7379$, evaluate $y(0.8)$ by Adam's predictor-corrector method.(N/D 2012)

9) Using Adam's method to find $y(2)$ if $y' = (x + y)/2$, $y(0) = 2$, $y(0.5) = 2.636$, $y(1) = 3.595$, $y(1.5) = 4.968$. (M/J 2013)

Unit – V (Boundary Value Problems in ODE and PDE)

• Finite Difference Solution of Second Order ODE

1) Solve the equation $y'' = x + y$ with the boundary conditions $y(0) = y(1) = 0$. (M/J 2012)

2) Solve $y'' - y = 0$ with the boundary conditions $y(0) = 0$ and $y(1) = 1$. (N/D 2012)

3) Solve, by finite difference method, the boundary value problem $y''(x) - y(x) = 0$, where $y(0) = 0$ and $y(1) = 1$, taking $h = 0.25$. (M/J 2012)

4) Using the finite difference method, compute $y(0.5)$, given $y'' - 64y + 10 = 0, x \in (0,1), y(0) = y(1) = 0$, subdividing the interval into (i) 4 equal parts (ii) 2 equal parts. (N/D 2011),(N/D 2012)

• One Dimensional Heat equation by explicit method

1) Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, subject to $u(0, t) = u(1, t) = 0$ and $u(x, 0) = \sin \pi x$, $0 < x < 1$, using Bender-Schmidt method. (M/J 2012)

2) Solve by Bender-Schmidt formula upto $t = 5$ for the equation $u_{xx} = u_t$, subject to $u(0, t) = 0$, $u(5, t) = 0$, and $u(x, 0) = x^2(25 - x^2)$, taking $h = 1$. (N/D 2012)

3) Solve $\frac{\partial u}{\partial t} = \frac{1}{2}\frac{\partial^2 u}{\partial x^2}$ with the condition $u(0, t) = 0 = u(4, t)$, $u(x, 0) = x(4 - x)$ taking $h = 1$ employing Bender-Schmidt recurrence equation. Continue the solution through 10 time steps.(M/J 2012)

4) Solve $u_{xx} = 32u_t$, $h = 0.25$ for $t \geq 0$, $0 < x < 1$, $u(0, t) = 0$, $u(x, 0) = 0$, $u(1, t) = t$. (M/J 2013)

• One Dimensional Heat equation by implicit method

1) Obtain the Crank – Nicholson finite difference method by taking $\lambda = \frac{kc^2}{h^2} = 1$. Hence, find $u(x, t)$ in the rod for two times steps for the heat equation, given $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, $u(x, 0) = \sin(\pi x), u(0, t) = 0, u(1, t) = 0$. Take h = 0.2.(A/M 2010)

• One Dimensional Wave equation

1) Solve the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, 0 < x < 1, t > 0$ satisfying the conditions $u(x, 0) = 0, \frac{\partial u}{\partial t}(x, 0) = 0, u(0, t) = 0$ and $u(1, t) = \frac{1}{2}\sin \pi t$. Compute $u(x, t)$ for 4 time-steps by taking $h = \frac{1}{4}$.(N/D 2012)

2) Find the pivotal values of the equation $\frac{1}{4}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ with given conditions $u(0, t) = 0, u(4, t) = 0, u(x, 0) = x(4 - x)$ and $\frac{\partial u}{\partial t}(x, 0) = 0$ by taking $h = 1$, for 4 time steps.(M/J 2012)

3) Solve $4u_{tt} = u_{xx}, u(0, t) = 0, u(4, t) = 0$, $u(x, 0) = x(4 - x), u_t(x, 0) = 0, h = 1$ upto $t = 4$. (M/J 2013)

• Two Dimensional Laplace and Poisson equations

1) Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown: (M/J 2012)

2) *Deduce the standard five point formula for $\nabla^2 u = 0$. Hence, solve it over the square region given by the boundary conditions as in the figure below:(A/M 2010)

3) Solve the Poisson equation $\nabla^2 u = -10\left(x^2 + y^2 + 10\right)$ over the square mesh with sides $x = 0, y = 0, x = 3$ and $y = 3$ with $u = 0$ on the boundary and mesh length 1 unit.

(N/D 2012)

4) Solve $\nabla^2 u = 8x^2y^2$ for square mesh given $u = 0$ on the four boundaries dividing the square into 16 sub-squares of length 1 unit. (N/D 2011)

5) Solve $\nabla^2 u = 8x^2y^2$ over the square $x = -2, x = 2, y = -2, y = 2$ with $u = 0$ on the boundary and mesh length = 1. (M/J 2013)

----*All the Best*----