SUBJECT NAME : Engineering Mathematics - I

SUBJECT CODE : MA2111

MATERIAL NAME : University Questions

MATERIAL CODE : SKMA00100

Name of the Student: Branch:

Unit – I (Matrices)

- **Cayley – Hamilton Theorem**

1. Find the characteristic equation of the matrix A given $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$. Hence find A^{-1} ande A^4. (Jan 2009)

2. Show that the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{bmatrix}$ satisfies the characteristics equation and hence find its inverse. (Jan 2011),(Jan 2013)

3. Using Cayley-Hamilton theorem, find the inverse of $A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$.(N/D 2011)(AUT)

4. Using Cayley – Hamilton theorem, find the inverse of the matrix $A = \begin{bmatrix} -1 & 0 & 3 \\ 8 & 1 & 7 \\ -3 & 0 & 8 \end{bmatrix}$. (N/D 2010)

5. Using Cayley – Hamilton theorem, find A^{-1} when $A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$. (M/J 2010)

6. Use Cayley – Hamilton theorem to find the value of the matrix given by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$, if the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$. (M/J 2009)

7. Verify Cayley Hamilton Theorem and hence find A^{-1} for $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$. (Jan 2010)

8. Verify Cayley Hamilton Theorem for the matrix $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 4 & -2 \\ -1 & 1 & 2 \end{pmatrix}$. (A/M 2011)

9. Verify Cayley Hamilton Theorem for the matrix $\begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$ and hence find A^{-1} and A^4. (M/J 2012)

10. Find A^n using Cayley Hamilton theorem, taking $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. Hence find A^3. (Jan 2012)

• Eigen Values and Eigen Vectors of a given matrix

1. Find the eigen values and eigen vectors of $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. (Jan 2009)

2. Find the eigenvalues and eigenvectors of $\begin{bmatrix} 2 & 1 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$. (Jan 2013)

3. Find all the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$. (Jan 2011)(AUT)

4. Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$. (N/D 2011)(AUT)

5. Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$. (M/J 2010),(N/D 2010),(Jan 2012)

6. Find the eigen values and eigen vectors for the matrix $A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$. (M/J 2009),(Jan 1010)

• Diagonalisation of a Matrix

1. The eigen vectors of a 3X3 real symmetric matrix A corresponding to the eigen-values $2, 3, 6$ are $(1, 0, -1)^T$, $(1, 1, 1)^T$ and $(1, 2, -1)^T$ respectively. Find the matrix A.

(A/M 2011)

2. If the eivenvalues of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ are 0, 3, 15, find the eigenvectors of A and diagonalize the matrix A. (Jan 2013)

• Quadratic form to Canonical form

1. Reduce the given quadratic form Q to its canonical form using orthogonal transformation. $Q = x^2 + 3y^2 + 3z^2 - 2yz$. (Jan 2009)

2. Reduce the quadratic form $2x^2 + 5y^2 + 3z^2 + 4xy$ to the Canonical form by orthogonal reduction and state its nature. (M/J 2010),(Jan 2012)

3. Reduce the quadratic form $2x_1x_2 + 2x_1x_3 - 2x_2x_3$ to a canonical form by an orthogonal reduction. Also find its nature. (A/M 2011)

4. Reduce the quadratic form $2x_1x_2 + 2x_2x_3 + 2x_3x_1$ into canonical form.(Jan 2013)

5. Reduce the quadratic form $2x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$ to canonical form by an orthogonal transformation. Also find the rank, index, signature and nature of the quadratic form.(N/D 2010)

6. Find a change of variables that reduces the quadratic form $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ to a sum of squares and express the quadratic form in terms of new variables. (Jan 2011)(AUT)

7. Reduce the quadratic form $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_3x_1$ into canonical form by means of an orthogonal transformation. (N/D 2011)(AUT)

8. Reduce the quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$ to the Canonical form through an orthogonal transformation and hence show that is positive semi definite. Also given a non – zero set of values (x_1, x_2, x_3) which makes this quadratic form zero. (M/J 2009)

9. Reduce the quadratic form $10x_1^2 + 2x_2^2 + 5x_3^2 + 6x_2x_3 - 10x_3x_1 - 4x_1x_2$ to a Canonical form through an orthogonal transformation and hence find rank, index, signature, nature and also give n0n – zero set of values for x_1, x_2, x_3 (if they exist), that will make the quadratic form zero. (Jan 2010)

10. Reduce the quadratic form $x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ to canonical form through an orthogonal transformation. Write down the transformation. (M/J 2012)

Unit – II (Three Dimensional Analytical Geometry)

• Sphere

1. Find the equation of the sphere passing through the points $(0, 0, 0)$, $(0, 1, -1)$, $(-1, 2, 0)$ and $(1, 2, 3)$. (N/D 2011)(AUT)

2. Obtain the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9$, $x + y + z = 3$ as a great circle. (Jan 2009)

3. Obtain the equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$, $x + y + z = 3$ as the greatest circle. (Jan 2012),(M/J 2012)

4. Find the equation to the sphere passing through the circle $x^2+y^2+z^2=9$, $x+y+z=1$ and cuts orthogonally the sphere $x^2+y^2+z^2+2x-4y-16z+17=0$. (M/J 2010)

5. Find the equation of the sphere passing through the circle $x^2+y^2+z^2+x-3y+2z-1=0,\ 2x+5y-z+7=0$ and cuts orthogonally the sphere $x^2+y^2+z^2-3x+5y-7z-6=0$. (N/D 2010)

6. Find the equation of the sphere having its centre on the plane $4x-5y-z=3$ and passing through the circle $x^2+y^2+z^2-2x-3y+4z+8=0$; $x-2y+z=8$. (N/D 2010)

7. Find the equation of the sphere described on the line joining the points $(2,-1,4)$ and $(-2,2,-2)$ as diameter. Find the area of the circle in which this sphere is cut by the plane $2x+y-z=3$. (Jan 2009)

8. Find the equation of the sphere of radius 3 and whose centre lies on the line $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z}{2}$ at a distance 2 from $(1,1,0)$. (A/M 2011)

9. Find the equations of the spheres which pass through the circle $x^2+y^2+z^2=5$ and $x+2y+3z=3$ and touch the plane $4x+3y=15$. (M/J 2009)

10. Find the equation of the sphere which passes through the circle $x^2+y^2+z^2-2x-4y=0$, $x+2y+3z=0$ and touch the plane $4x+3y=25$. (Jan 2011)(AUT)

11. Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z=3$ and find also the point of contact. (M/J 2009), (N/D 2011),(Jan 2013)

12. Find the two tangent planes to the sphere $x^2+y^2+z^2-4x+2y-6z+5=0$, which are parallel to the plane $x+4y+8z=0$. Find their point of contact. (Jan 2010),(M/J 2012),(Jan 2013)

13. Obtain the equation of the tangent planes to the sphere $x^2+y^2+z^2+2x-4y+6z-7=0$, which intersect in the line $6x-3y-23=0=3z+2$. (Jan 2012)

14. Find the equation of the tangent lines to the circle $3x^2+3y^2+3z^2-2x-3y-4z-22=0$, $3x+4y+5z-26=0$ at the point $(1,2,3)$.(Jan 2011)(AUT)

15. Find the centre, radius and area of the circle given by $x^2+y^2+z^2+2x-2y-4z-19=0$, $x+2y+2z+7=0$. (Jan 2010),(M/J 2010)

- **Cone**

1. Find the equation of the right circular cone whose vertex is at the origin and axis is the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and which has semi vertical angle of 30°. (Jan 2009),(N/D 2010)

2. Find the equation of the right circular cone whose vertex is $(2,1,0)$, semi vertical angle is 30° and the axis is the line $\frac{x-2}{3}=\frac{y-1}{1}=\frac{z}{2}$. (Jan 2013)

3. Find the equation of the right circular cone generated by revolving the line $x=0,\ y-z=0$ about the axis $x=0,\ z=2$.(M/J 2009)

4. Find the equation of the right circular cone generated when the straight line which is the intersection of the planes $2y+3z=6$ and $x=0$ revolves about the $z-$axis with constant angle. (Jan 2011)(AUT),(M/J 2012)

5. Find the equation of the cone whose vertex is $(1,2,3)$ and whose guiding curve is the circle $x^2+y^2+z^2=4,\ x+y+z=1$. (M/J 2009), (N/D 2011)(AUT)

6. Find the equation of the cone with vertex at $(1,1,1)$ and passing through curve of intersection of $x^2+y^2+z^2=1$ and $x+y+z=1$. (A/M 2011)

7. Find the equation of the cone formed by rotating the line $2x+3y=5,\ z=0$ about the y – axis. (Jan 2010)

8. Find the equation of the cone whose vertex is the point $(1,1,0)$ and whose base is the curve $y=0,\ x^2+z^2=4$. (M/J 2010)

9. Find the equation of the cone formed by rotating the line $2x+3y=6,\ z=0$ about the y – axis. (Jan 2012)

- **Cylinder**

1. Find the equation of the right circular cylinder whose axis is the line $x = 2y = -z$ and radius 4. (Jan 2009)

2. Find the equation of the right circular cylinder of radius 3 and axis $\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}$. (Jan 2010),(M/J 2010),(A/M 2011),(M/J 2012)

3. Find the equation of the right circular cylinder whose axis is $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$ and radius 2. (N/D 2010),(Jan 2012)

4. Find the equation of the right circular cylinder of radius 5 whose axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$. (N/D 2011)(AUT)

5. Find the equation of the right circular cylinder which passes through the circle $x^2 + y^2 + z^2 = 9,\ x - y + z = 3$. (Jan 2011)(AUT)

6. Find the equation of the cylinder whose generators are parallel to $\frac{x}{2} = \frac{y}{2} = \frac{z}{-3}$ and whose guiding curve is the ellipse $3x^2 + y^2 = 3$. (Jan 2013)

Unit – III (Differential Calculus)

• Radius of Curvature and Circle of curvature

1. Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$. (Jan 2009)

2. Find the circle of curvature at $\left(\frac{a}{4}, \frac{a}{4}\right)$ on $\sqrt{x} + \sqrt{y} = \sqrt{a}$.
(M/J 2010),(N/D 2010),(A/M 2011), (N/D 2011)(AUT),(Jan 2012),(M/J 2012)

3. Find the equation of circle of curvature of the parabola $y^2 = 12x$ at the point $(3, 6)$. (Jan 2009)

4. Find the equation of circle of curvature of the rectangular hyperbola $xy = 12$ at the point $(3, 4)$. (Jan 2010)

5. Find the radius of curvature at the point $(0, c)$ on the curve $y = c\cosh\left(\frac{x}{c}\right)$ (M/J 2009)

6. Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy$. (N/D 2011)(AUT)

7. Find the radius of curvature at the point $(a\cos^3\theta, a\sin^3\theta)$ on the curve $x^{2/3} + y^{2/3} = a^{2/3}$. (M/J 2009)

8. Find the radius of curvature at $(a, 0)$ on $y^2 = \frac{a^3 - x^3}{x}$. (Jan 2010)

9. Prove that the radius of curvature of the curve $xy^2 = a^3 - x^3$ at the point $(a, 0)$ is $\frac{3a}{2}$. (N/D 2010)

10. Find the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$. (M/J 2010),(M/J 2012),(Jan 2013)

11. Find the radius of curvature of the curve $x = 3a\cos\theta - a\cos 3\theta$, $y = 3a\sin\theta - a\sin 3\theta$. (A/M 2011)

12. If $y = \frac{ax}{a+x}$, prove that $\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$, where ρ is the radius of curvature. (Jan 2012)

- **Evolute**

1. Show that the evolute of the parabola $y^2 = 4ax$ is the curve $27ay^2 = 4(x - 2a)^3$. (Jan 2010),(M/J 2010)

2. Find the equation of the evolute of the parabola $y^2 = 4ax$. (Jan 2011)(AUT),(Jan 2012),(M/J 2012)

3. Find the radius of curvature and centre of curvature of the parabola $y^2 = 4ax$ at the point t. Also find the equation of the evolute. (Jan 2013)

4. Find the evolute of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. (N/D 2010),(N/D 2011)(AUT)

5. Obtain the equation of the evolute of the curve $x=a(\cos\theta+\theta\sin\theta)$, $y=a(\sin\theta-\theta\cos\theta)$. (M/J 2009)

6. Show that the evolute of the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ is another cycloid. (A/M 2011)

• Envelope

1. Find the envelope of the family of straight lines $x\cos\alpha+y\sin\alpha=c\sin\alpha\cos\alpha$, α being the parameter. (A/M 2011)

2. Find the envelope of the straight line $\frac{x}{a}+\frac{y}{b}=1$, where a and b are parameters that are connected by the relation $a+b=c$. (Jan 2009),(M/J 2009)

3. Find the envelope of $\frac{x}{a}+\frac{y}{b}=1$, where a and b are connected by the relation $a^2+b^2=c^2$, c being constant. (N/D 2010),(Jan 2013)

4. Find the envelope of the straight line $\frac{x}{a}+\frac{y}{b}=1$ where the parameters a and b are connected by the relation $a^n+b^n=c^n$, c being a constant. (N/D 2011)(AUT)

5. Find the envelope of $\frac{x}{l}+\frac{y}{m}=1$, where the parameters l and m are connected by the relation $\frac{l}{a}+\frac{m}{b}=1$ (a and b are constants). (Jan 2012)

6. Find the envelope of the straight line $\frac{x}{a}+\frac{y}{b}=1$, where a and b are connected by the relation $ab=c^2$, c is a constant. (Jan 2010),(M/J 2010)

7. Find the envelope of the system of ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a and b are connected by the relation $ab=4$. (M/J 2012)

8. Find the envelope of the circles drawn upon the radius vectors of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ as diameter. (Jan 2013)

• Evolute as the envelope of normals

1. Find the evolute of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ considering it as the envelope of its normals. (Jan 2009)

Unit – IV (Functions of several variables)

• Euler's Theorem

1. If $u = x^y$, show that $u_{xxy}=u_{xyx}$. (Jan 2009)

2. If $u = \log\left(x^2+y^2\right)+\tan^{-1}(y/x)$ prove that $u_{xx}+u_{yy}=0$.

   (Jan 2009),(N/D 2010)

3. If $u=\cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=-\frac{1}{2}\cot u$. (N/D 2011)(AUT)

4. If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{-\sin u\cos 2u}{4\cos^3 u}$

   (A/M 2011)

5. If $u=\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that (1) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$ and (2) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\tan^2 u$. (Jan 2011)(AUT)

• Total derivatives

1. If $u=f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$. (M/J 2009)

2. If $z = f(x, y)$, where $x = u^2 - v^2$, $y = 2uv$, prove that

$$\frac{\partial^2 z}{\partial u^2}+\frac{\partial^2 z}{\partial v^2} = 4(u^2+v^2)\left(\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}\right).$$

(Jan 2010),(Jan 2012)

3. If $x = u\cos\alpha - v\sin\alpha$, $y = u\sin\alpha + v\cos\alpha$ and $V = f(x, y)$, show that

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=\frac{\partial^2 V}{\partial u^2}+\frac{\partial^2 V}{\partial v^2}.$$

(Jan 2011)(AUT)

4. If $u = e^{xy}$, show that

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{1}{u}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right].$$

(Jan 2013)

5. If F is a function of x and y and if $x = e^u \sin v$, $y = e^u \cos v$, prove that

$$\frac{\partial^2 F}{\partial x^2}+\frac{\partial^2 F}{\partial y^2}=e^{-2u}\left[\frac{\partial^2 F}{\partial u^2}+\frac{\partial^2 F}{\partial v^2}\right].$$

(Jan 2013)

6. If $u = f(x, y)$ where $x = r\cos\theta$, $y = r\sin\theta$, prove that

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2=\left(\frac{\partial u}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2.$$

(M/J 2010)

7. If $u = x^2 + y^2 + z^2$ and $x = e^{2t}$, $y = e^{2t}\cos 3t$, $z = e^{2t}\sin 3t$, Find $\frac{du}{dt}$.

(N/D 2011)(AUT)

• Taylor's expansion

1. Find the Taylor series expansion of $e^x \sin y$ at the point $(-1, \pi/4)$ up to 3rd degree terms. (Jan 2009),(M/J 2009)

2. Find the Taylor's series expansion of $e^x \cos y$ in the neighborhood of the point $\left(1, \frac{\pi}{4}\right)$ upto third degree terms. (N/D 2010)

3. Expand $e^x \log(1 + y)$ in power of x and y upto terms of third degree using Taylor's theorem. (N/D 2011)(AUT)

4. Find the Taylor's series expansion of $x^2y^2 + 2x^2y + 3xy^2$ in powers of $(x + 2)$ and $(y - 1)$ upto 3rd degree terms. (Jan 2010),(M/J 2010),(Jan 2012)

5. Use Taylor's formula to expand the function defined by $f(x, y) = x^3 + y^3 + xy^2$ in powers of $(x-1)$ and $(y-2)$. (A/M 2011)

6. Expand $x^2 y + 3y - 2$ in powers of $(x-1)$ and $(y+2)$ upto 3rd degree terms. (M/J 2012)

• Maxima and Minima

1. Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.
(Jan 2010),(A/M 2011),(Jan 2012)

2. Find the maximum and minimum values of $x^2 - xy + y^2 - 2x + y$. (M/J 2012)

3. Discuss the maxima and minima of the function $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$. (N/D 2010)

4. Test for an extrema of the function $f(x, y) = x^4 + y^4 - x^2 - y^2 - 1$. (Jan 2011)(AUT)

5. Examine the function $f(x, y) = x^3 y^2 (12 - x - y)$ for extreme values. (M/J 2009)

6. Test for the maxima and minima of the function $f(x, y) = x^3 y^2 (6 - x - y)$. (Jan 2013)

7. Find the maximum value of $x^m y^n z^p$ subject to the condition $x + y + z = a$. (Jan 2009)

8. A rectangular box open at the top, is to have a volume of 32 cc. Find the dimensions of the box, that requires the least material for its construction.
(M/J 2010),(N/D 2011)(AUT),(M/J 2012)

9. Find the volume of the greatest rectangular parallelepiped inscribed in the ellipsoid whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. (M/J 2009)

10. If $x^2 + y^2 + z^2 = r^2$, show that the maximum value of $yz + zx + xy$ is r^2 and the minimum value is $-\frac{r^2}{2}$. (Jan 2013)

• Jacobians

1. Find the Jacobian $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$ of the transformation $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$. (Jan 2009),(A/M 2011)

2. If $x + y + z = u$, $y + z = uv$, $z = uvw$ prove that $\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2 v$. (Jan 2010),(Jan 2012)

3. Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 if $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_3 x_1}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$. (N/D 2010)

Unit – V (Multiple Integrals)

• Simple problems on double integral

No recent problem from this topic

• Change of order of integration

1. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \, dxdy$ by changing the order of integration. (N/D 2010),(A/M 2011)

2. Change the order of integration in $\int_0^a \int_{a-y}^{\sqrt{a^2-y^2}} y \, dxdy$ and then evaluate it. (M/J 2009)

3. Change the order of integration $\int_0^1 \int_{x^2}^{2-x} xy \, dxdy$ and hence evaluate.

(Jan 2010),(M/J 2012)

4. Change the order of integration in the interval $\int_0^a \int_{x^2/a}^{2a-x} xy \, dydx$ and hence evaluate it.

(M/J 2010),(Jan 2013)

5. Change the order of integration and hence find the value of $\int_0^1 \int_y^{2-y} xy \, dxdy$.

(N/D 2011)(AUT)

6. Change the order of integration and hence evaluate $\int_1^3 \int_{y=0}^{6/x} x^2 \, dydx$. (Jan 2009)

7. Change the order of integration $\int_0^{2a}\int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} xy\,dxdy$ and hence evaluate it.

(Jan 2011)(AUT)

8. Change the order of integration in $\int_0^{a}\int_0^{\frac{b}{a}\sqrt{a^2-x^2}} x^2\,dydx$ and then evaluate it.

(Jan 2012)

• Change into polar coordinates

1. Express $\int_0^a\int_y^a \frac{x^2\,dxdy}{\left(x^2+y^2\right)^{3/2}}$ in polar coordinates and then evaluate it. (M/J 2009)

2. Evaluate $\int_0^\infty\int_0^\infty e^{-\left(x^2+y^2\right)}\,dxdy$ by converting to polar coordinates. Hence deduce the value of $\int_0^\infty e^{-x^2}dx$. (Jan 2010),(N/D 2010)

3. Transform the integral $\int_0^2\int_0^{\sqrt{2x-x^2}}\left(x^2+y^2\right)dydx$ into polar coordinates and hence evaluate it. (A/M 2011)

4. By Transforming into polar coordinates, evaluate $\iint\left(\frac{x^2y^2}{x^2+y^2}\right)dxdy$ over annular region between the circles $x^2+y^2=16$ and $x^2+y^2=4$. (M/J 2010)

5. By Transforming into polar coordinates, evaluate $\iint \frac{x^2y^2}{x^2+y^2}\,dxdy$ over annular region between the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$, $(b>a)$. (Jan 2013)

6. Transform the double integral $\int_0^a\int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}}\frac{dxdy}{\sqrt{a^2-x^2-y^2}}$ into polar co-ordinates and then evaluate it. (Jan 2012)

7. Transform the integral into polar coordinates and hence evaluate

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\,dydx.$$ (Jan 2012)

• Area as a double integral

1. Find the area bounded by the parabolas $y^2 = 4 - x$ and $y^2 = x$ by double integration.

(N/D 2010)

2. Find, by double integration, the area enclosed by the curves $y^2 = 4ax$ and $x^2 = 4ay$.

(Jan 2010),(A/M 2011)

3. Find, by double integration, the area between the two parabolas $3y^2 = 25x$ and $5x^2 = 9y$. (M/J 2012)

4. Find the area common to $y^2 = 4x$ and $x^2 = 4y$ using double integration.

(N/D 2011)(AUT)

5. Evaluate $\iint (x-y)\,dxdy$ over the region between the line $y = x$ and the parabola $y = x^2$. (Jan 2011)(AUT)

6. Find the smaller of the areas bounded by the ellipse $4x^2 + 9y^2 = 36$ and the straight line $2x + 3y = 6$. (Jan 2012)

7. Find the area inside the circle $r = a\sin\theta$ but lying outside the cardioids $r = a(1-\cos\theta)$. (Jan 2009)

8. Find the area which is inside the circle $r = 3a\cos\theta$ and outside the cardioids $r = a(1+\cos\theta)$. (Jan 2013)

9. Evaluate $\int_C \left[(3x^2 + y^3)\,dx + (x^3 + 3xy^2)\,dy \right]$ where C is the parabola $y^2 = 4ax$ from $(0, 0)$ to $(a, 2a)$. (M/J 2009)

• Triple integral

1. Evaluate $\int_0^a \int_0^b \int_0^c (x^2+y^2+z^2)\,dxdydz$. (Jan 2009)

2. Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+y} e^{x+y+z}\,dxdydz$. (M/J 2009)

3. Evaluate $\int_0^{a}\int_0^{\sqrt{a^2-x^2}}\int_0^{\sqrt{a^2-x^2-y^2}} \frac{1}{\sqrt{a^2-x^2-y^2-z^2}}\,dzdydx$. (N/D 2011)(AUT)

4. Evaluate $\int_0^{1}\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$. (Jan 2012),(Jan 2013)

5. Using triple integration, find the volume of the sphere $x^2+y^2+z^2=a^2$. (N/D 2010)

6. Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. (Jan 2010),(A/M 2011)

7. Find the volume of the tetrahedran bounded by the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ and the coordinate plane $x=0, y=0, z=0$. (M/J 2010)

8. Evaluate $\iiint x^2yz\,dxdydz$ taken over the tetrahedron bounded by the planes $x=0, y=0, z=0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. (Jan 2011)(AUT)

9. Change to spherical polar co – ordinates and hence evaluate $\iiint_V \frac{1}{\sqrt{x^2+y^2+z^2}}\,dxdydz$, where V is the volume of the sphere $x^2+y^2+z^2=a^2$. (Jan 2009)

10. Find the value of $\int\int\int xyz\,dxdydz$ through the positive spherical octant for which $x^2+y^2+z^2\le a^2$. (M/J 2010)

11. Evaluate $\iiint \frac{dzdydx}{(x+y+z+1)^3}$ where V is the region bounded by $x=0, y=0, z=0, x+y+z=1$. (N/D 2011)(AUT)

----All the Best----