SUBJECT NAME : **Discrete Mathematics**

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MATERIAL NAME : **University Questions**

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Name of the Student: Branch:

Unit – I (Logic and Proofs)

• Simplification by Truth Table and without Truth Table

1. Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent. (N/D 2012)

2. Without using the truth table, prove that $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$.
(N/D 2010)

3. Prove that $(P \rightarrow Q) \wedge (R \rightarrow Q) \Rightarrow (P \vee R) \rightarrow Q$. (M/J 2013)

• PCNF and PDNF

4. Without using truth table find the PCNF and PDNF of
$P \rightarrow (Q \wedge P) \wedge (\neg P \rightarrow (\neg Q \wedge \neg R))$. (A/M 2011)

5. Find the principal disjunctive normal form of the statement,
$(q \vee (p \wedge r)) \wedge ((p \vee r) \wedge q)$. (N/D 2012)

6. Obtain the principal disjunctive normal form and principal conjunction form of the statement $p \vee \neg p (\rightarrow (q \vee (\neg q \rightarrow r)) .)$ (N/D 2010)

7. Show that $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P) = (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge$
$(P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$. (M/J 2013)

• Theory of Inference

8. Show that: $(P \to Q) \wedge (R \to S)$, $(Q \wedge M) \wedge (S \to N)$, $\neg (M \wedge N)$ and $(P \to R) \Rightarrow \neg P$. (A/M 2011)

9. Prove that the following argument is valid: $p \to \neg q, r \to q, r \Rightarrow \neg p$.

(M/J 2012)

10. Prove that the premises $a \to (b \to c)$, $d \to (b \wedge \neg c)$ and $(a \wedge d)$ are inconsistent. (N/D 2010)

11. Using indirect method of proof, derive $p \to \neg s$ from the premises $p \to (q \vee r)$, $q \to \neg p$, $s \to \neg r$ and p. (N/D 2011)

12. Show that the hypothesis, "It is not sunny this afternoon and it is colder than yesterday", "we will go swimming only if it is sunny", "If we do not go swimming, then we will take a canoe trip" and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset".(N/D 2012)

13. Determine the validity of the following argument:

If 7 is less than 4, then 7 is not a prime number, 7 is not less than 4. Therefore 7 is a prime number.(M/J 2012)

14. Prove that 2 is irrational by giving a proof using contradiction. (N/D 2011),(M/J 2013)

• Quantifiers

15. Show that $(\forall x)(P(x) \to Q(x))$, $(\exists y) P(y) \Rightarrow (\exists x) Q(x)$. (M/J 2012)

16. Use the indirect method to prove that the conclusion $\exists z Q(z)$ follows form the premises $\forall x (P(x) \to Q(x))$ and $\exists y P(y)$. (N/D 2012)

17. Prove that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x) P(x) \vee (\exists x) Q(x)$. (M/J 2013)

18. Prove that $\forall x (P(x) \to Q(x))$, $\forall x (R(x) \to \neg Q(x)) \Rightarrow \forall x (R(x) \to \neg P(x))$.

(N/D 2010)

19. Use indirect method of proof to prove that
$(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$. (A/M 2011),(N/D 2011)

20. Show that the statement "Every positive integer is the sum of the squares of three integers" is false.(N/D 2011)

21. Verify the validity of the following argument. Every living thing is a plant or an animal. John's gold fish is alive and it is not a plant. All animals have hearts. Therefore John's gold fish has a heart.(M/J 2012)

22. Verify that validating of the following inference.

If one person is more successful than another, then he has worked harder to deserve success. Ram has not worked harder than Siva. Therefore, Ram is not more successful than Siva.(A/M 2011)

Unit – II (Combinatorics)

• Mathematical Induction and Strong Induction

1. Prove by the principle of mathematical induction, for 'n' a positive integer,

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$ (M/J 2012)

2. Use Mathematical induction show that $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$. (A/M 2011)

3. Using mathematical induction to show that
$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} > n,\ n \geq 2.$$ (N/D 2011)

4. Prove, by mathematical induction, that for all $n \geq 1$, $n^3 + 2n$ is a multiple of 3.

(N/D 2010)

5. Use Mathematical induction to prove the inequality $n < 2^n$ for all positive integer n.(N/D 2012)

6. Let m any odd positive integer. Then prove that there exists a positive integer n such that m divides $2^n - 1$. (M/J 2013)

7. State the Strong Induction (the second principle of mathematical induction). Prove that a positive integer greater than 1 is either a prime number or it can be written as product of prime numbers.(M/J 2013)

• Pigeonhole Principle

8. If n Pigeonholes are occupied by $(kn+1)$ pigeons, where k is positive integer, prove that at least one Pigeonhole is occupied by $k+1$ or more Pigeons. Hence, find the minimum number of m integers to be selected from $S = \{1, 2, ..., 9\}$ so that the sum of two of the m integers are even. (N/D 2011)

9. What is the maximum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade if there are five possible grades A, B, C, D and F?(N/D 2012)

• Permutations and Combinations

10. How many positive integers n can be formed using the digits 3,4,4,5,5,6,7 if n has to exceed 5000000?(N/D 2010)

11. Find the number of distinct permutations that can be formed from all the letters of each word (1) RADAR (2) UNUSUAL. (M/J 2012)

12. A box contains six white balls and five red balls. Find the number of ways four balls can be drawn from the box if

    (1) They can be any colour

    (2) Two must be white and two red

    (3) They must all be the same colour (A/M 2011)

• Solving recurrence relations by generating function

13. Using the generating function, solve the difference equation $y_{n+2} - y_{n+1} - 6y_n = 0,\ y_1 = 1,\ y_0 = 2$. (N/D 2010)

14. Using generating function solve $y_{n+2} - 5y_{n+1} + 6y_n = 0,\ n \ge 0$ with $y_0 = 1$ and $y_1 = 1$. (A/M 2011)

15. Using method of generating function to solve the recurrence relation
$a_n = 4a_{n-1} - 4a_{n-2} + 4^n$; $n \geq 2$, given that $a_0 = 2$ and $a_1 = 8$. (N/D 2011)

16. Use generating functions to solve the recurrence relation
$a_n + 3a_{n-1} - 4a_{n-2} = 0, n \geq 2$ with the initial condition $a_0 = 3, a_1 = -2$.

(N/D 2012)

17. Using generating function, solve the recurrence relation $a_n - 5a_{n-1} + 6a_{n-2} = 0$
where $n \geq 2, a_0 = 0$ and $a_1 = 1$. (M/J 2013)

18. Solve the recurrence relation $a_{n+1} - a_n = 3n^2 - n, n \geq 0, a_0 = 3$. (N/D 2011)

19. Solve the recurrence relation, $S(n) = S(n-1) + 2(n-1)$, with
$S(0) = 3, S(1) = 1$, by finding its generating function. (M/J 2012)

- **Inclusion and Exclusion**

20. Find the number of integers between 1 and 250 both inclusive that are divisible by any of the integers 2,3,5,7.(N/D 2010)

21. Determine the number of positive integers $n, 1 \leq n \leq 2000$ that are not
divisible by 2,3 or 5 but are divisible by 7. (M/J 2013)

22. There are 2500 students in a college, of these 1700 have taken a course in C, 1000 have taken a course Pascal and 550 have taken a course in Networking. Further 750 have taken courses in both C and Pascal. 400 have taken courses in both C and Networking, and 275 have taken courses in both Pascal and Networking. If 200 of these students have taken courses in C, Pascal and Networking.

(1) How many of these 2500 students have taken a course in any of these three courses C, Pascal and Networking?

(2) How many of these 2500 students have not taken a course in any of these three courses C, Pascal and Networking?(A/M 2011)

23. Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members form the mathematics department and four from the computer science department?(N/D 2012)

Unit – III (Graph Theory)

• Drawing graphs from given conditions

1. Draw the complete graph K_5 with vertices A, B, C, D, E. Draw all complete sub graph of K_5 with 4 vertices. (N/D 2010)

2. Draw the graph with 5 vertices, A, B, C, D, E such that $\deg(A) = 3$, B is an odd vertex, $\deg(C) = 2$ and D and E are adjacent. (N/D 2010)

3. Draw the graph with 5 vertices A, B, C, D and E such that $\deg(A) = 3$, B is an odd vertex, $\deg(C) = 2$ and D and E are adjacent. (A/M 2011)

4. Determine which of the following graphs are bipartite and which are not. If a graph is bipartite, state if it is completely bipartite.(N/D 2011)

5. Find the all the connected sub graph obtained from the graph given in the following Figure, by deleting each vertex. List out the simple paths from A to in each sub graph.(A/M 2011)

• Isomorphism of graphs

6. Determine whether the graphs G and H given below are isomorphic. (N/D 2012)

7. Using circuits, examine whether the following pairs of graphs G_1 , G_2 given below are isomorphic or not: (N/D 2011)

8. Examine whether the following pair of graphs are isomorphic. If not isomorphic, give the reasons.(A/M 2011)

9. Check whether the two graphs given are isomorphic or not. (M/J 2013)

10. The adjacency matrices of two pairs of graph as given below. Examine the isomorphism of G and H by finding a permutation matrix.

$$A_G = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, A_H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

(N/D 2010)

- **General problems in graphs**

11. How many paths of length four are there from a and d in the simple graph G given below.(N/D 2012)

12. Find an Euler path or an Euler circuit, if it exists in each of the three graphs below. If it does not exist, explain why?(N/D 2011)

13. Check whether the graph given below is Hamiltonian or Eulerian or 2-colorable. Justify your answer.(M/J 2013)

• Theorems

14. Prove that an undirected graph has an even number of vertices of odd degree. (N/D 2012)

15. Show that if a graph with n vertices is self-complementary then $n \equiv 0 \text{ or } 1 \pmod 4$. (M/J 2013)

16. Prove that the maximum number of edges in a simple disconnected graph G with n vertices and k components is $\frac{(n-k)(n-k+1)}{2}$. (N/D 2011)

17. Show that graph G is disconnected if and only if its vertex set V can be partitioned into two nonempty subsets V_1 and V_2 such that there exists no edge is G whose one end vertex is in V_1 and the other in V_2. (M/J 2012)

18. If all the vertices of an undirected graph are each of degree k, show that the number of edges of the graph is a multiple of k. (N/D 2010)

19. Let G be a simple undirected graph with adjacency matrix A with respect to the ordering $v_1, v_2, v_3, \ldots, v_n$. Prove that the number of different walks of length r from v_i to v_j equals the (i, j) th entry of A^r, where r is a positive integer. (M/J 2013)

• Theorems based on Euler and Hamilton graph

20. Prove that a connected graph G is Eulerian if and only if all the vertices are on even degree. (M/J 2012)

21. Show that the complete graph with n vertices K_n has a Hamiltonian circuit whenever $n \geq 3$. (N/D 2012)

22. Prove that if G is a simple graph with at least three vertices and $\delta(G) \geq \frac{V(G)}{2}$ then G is Hamiltonian. (M/J 2013)

23. Let G be a simple undirected graph with n vertices. Let u and v be two non adjacent vertices in G such that $\deg(u) + \deg(v) \geq n$ in G. Show that G is Hamiltonian if and only if $G + uv$ is Hamiltonian. (A/M 2011)

Unit – IV (Algebraic Structures)

• Group, Subgroup and Normal Subgroup

1. If $(G,*)$ is an abelian group, show that $(a*b)^2 = a^2 * b^2$. (N/D 2010)

2. If * is a binary operation on the set R of real numbers defined by $a*b = a+b+2ab$,

   (1) Find $R,*$ is a semigroup

   (2) Find the identity element if it exists

   (3) Which elements has inverse and what are they? (A/M 2011)

3. State and prove Lagrange's theorem. (N/D 2010),(A/M 2011),(M/J 2012)

4. Prove that the order of a subgroup of a finite group divides the order of the group.(N/D 2011),(M/J 2013)

5. Prove the theorem: Let $G,*$ be a finite cyclic group generated by an element $a \in G$. If G is of order n, that is, $G = n$, then $a^n = e$, so that $G = \{a, a^2, a^3, ..., a^n = e\}$. Further more n is a least positive integer for which $a^n = e$. (N/D 2011)

6. Prove that intersection of two normal subgroups of a group $(G,*)$ is a normal subgroup of a group $(G,*)$. (M/J 2013)

7. Prove that the necessary and sufficient condition for a non empty subset H of a group $\{G,*\}$ to be a sub group is $a, b \in H \Rightarrow a*b^{-1} \in H$. (N/D 2012)

8. If * is the operation defined on $S = Q \times Q$, the set of ordered pairs of rational numbers and given by $(a,b)*(x,y) = (ax, ay+b)$, show that $(S,*)$ is a semi group. Is it commutative? Also find the identity element of S. (N/D 2012)

9. Define the Dihedral group $D_4,*$ and give its composition table. Hence find the identify element and inverse of each element. (A/M 2011)

• Homomorphism and Isomorphism

10. Prove that every finite group of order n is isomorphic to a permutation group of order n .(N/D 2011),(M/J 2013)

11. Let $f : G \to G'$ be a homomorphism of groups with Kernel K . Then prove that

K is a normal subgroup of G and G / K is isomorphic to the image of f .

(M/J 2012)

12. Let $(G,*)$ and (H,Δ) be two groups and $g : (G,*) \to (H,\Delta)$ be group homomorphism. Then prove that the Kernel of g is normal subgroup of $(G,*)$.

(M/J 2013)

13. Show that the Kernel of a homomorphism of a group $G,*$ into an another group H,Δ is a subgroup of G . (A/M 2011)

14. If $f : G \to G'$ is a group homomorphism from $\{G,*\}$ to $\{G',\Delta\}$ then prove that for any $a \in G$, $f(a^{-1}) = [f(a)]^{-1}$. (N/D 2012)

15. If $(Z,+)$ and $(E,+)$ where Z is the set all integers and E is the set all even integers, show that the two semi groups $(Z,+)$ and $(E,+)$ are isomorphic.

(N/D 2010)

16. Let $(S,*)$ be a semigroup. Then prove that there exists a homomorphism $g : S \to S^S$, where S^S, is a semigroup of functions from S to S under the operation of (left) composition. (N/D 2011)

• Ring and Fields

17. Show that $(Z,+,\times)$ is an integral domain where Z is the set of all integers.

(N/D 2010)

18. Prove that the set $Z_4 = \{[0],[1],[2],[3]\}$ is a commutative ring with respect to the binary operation addition modulo and multiplication modulo $+_4, \times_4$.

(N/D 2012)

Unit – V (Lattices and Boolean algebra)

• Partially Ordered Set (Poset)

1. Show that $(N, \leq)$ is a partially ordered set where N is set of all positive integers and $\leq$ is defined by $m \leq n$ iff $n - m$ is a non-negative integer. (N/D 2010)

2. Draw the Hasse diagram for (1) $P_1 = \{2, 3, 6, 12, 24\}$ (2) $P_2 = \{1, 2, 3, 4, 6, 12\}$ and $\leq$ is a relation such $x \leq y$ if and only is $x \mid y$. (A/M 2011)

3. Draw the Hasse diagram representing the partial ordering $\{(A, B) : A \subseteq B\}$ on the power set $P(S)$ where $S = \{a, b, c\}$. Find the maximal, minimal, greatest and least elements of the poset. (N/D 2012)

• Lattices

4. Let L be lattice, where $a * b = \text{glb}(a,b)$ and $a \oplus b = \text{lub}(a,b)$ for all $a, b \in L$. Then both binary operations $*$ and $\oplus$ defined as in L satisfies commutative law, associative law, absorption law and idempotent law.(M/J 2013)

5. In a distributive Lattice $\{L, \vee, \wedge\}$ if an element $a \in L$ a complement then it is unique. (N/D 2012)

6. Show that every chain is a lattice. (M/J 2013)

7. Prove that every distributive lattice is modular. Is the converse true? Justify your claim.(A/M 2011)

8. Show that in a distributive and complemented lattice $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$. (M/J 2013)

9. Show that the direct product of any two distributive lattices is a distributive lattice.(A/M 2011) ,(M/J 2012)

10. If $P(S)$ is the power set of a set S and $\cup, \cap$ are taken as join and meet, prove that $P(S), \subseteq$ is a lattice. Also, prove the modular inequality of a Lattice $L, \leq$ for any $a, b, c \in L$, $a \leq c \Leftrightarrow a \vee (b \wedge c) \leq (a \vee b) \wedge c$. (N/D 2011)

11. Prove that Demorgan's laws hold good for a complemented distributive lattice

$L, \wedge, \vee$, viz $(a \vee b)' = a' \wedge b'$ and $(a \wedge b)' = a' \vee b'$. (N/D 2011),(M/J 2013)

12. If S_{42} is the set all divisors of 42 and D is the relation "divisor of" on S_{42}, prove

that $\{S_{42}, D\}$ is a complemented Lattice. (N/D 2010)

- **Boolean Algebra**

13. In a Boolean algebra, prove that $(a \wedge b)' = a' \vee b'$. (N/D 2010)

14. In any Boolean algebra, show that $ab' + a'b = 0$ if and only if $a = b$.(N/D 2011)

15. In any Boolean algebra, prove that the following statements are equivalent:

(1) $a+b=b$
(2) $a\,b=a$
(3) $a' + b = 1$ and
(4) $a\,b' = 0$

(N/D 2011)

16. In a Boolean algebra, prove that $a.(a+b) = a$, for all $a, b \in B$. (N/D 2012)

17. Simplify the Bookean expression $a'.b'.c + a.b'.c + a'.b'.c'$ using Boolean algebra identities.(N/D 2012)

18. Let B be a finite Boolean algebra and let A be the set of all atoms of B. Then prove that the Boolean algebra B is isomorphic to the Boolean algebra $P(A)$,

where $P(A)$ is the power set of A. (M/J 2012)

19. Prove that D_{110}, the set of all positive divisors of a positive integer 110, is a

Boolean algebra and find all its sub algebras. (A/M 2011)

----*All the Best*----